2.3 Product & Quotient Rules and Higher Order Derivatives

What will you learn?

- **Find the derivative using the Product Rule**
- **Find the derivative using the Quotient Rule**
- **Find the derivative of a trig function**
- **Find a higher-order derivative of a function**

The Product Rule

The product of two differentiable function *f* and *g* is itself differentiable. Moreover, the derivative of *fg* is the first function times the derivative of the second, plus the second function times the derivative of the first.

$$\frac{d}{dx}[\, f(x)g(x) \,] = f(x)\, g'(x) + g(x)\, f'(x)$$

Example 1 - Using the Product Rule

Find the derivative of $h(x) = (3x - 2x^2)(5 + 4x)$

Example 2 - Using the Product Rule

Find the derivative of

$$y = 3x^2 \sin x$$

Example 3 - Using the Product Rule

Find the derivative of

$$y = 2x \cos x - 2 \sin x$$

Quotient Rule

The quotient *f/g* of two differentiable function *f* and *g* is itself differentiable at all values of x for which $g(x) \neq 0$.
Moreover, the derivative of *f/g* is given by the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator, all divided by the square of the denominator.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\,f'(x) - f(x)\,g'(x)}{[g(x)]^2} \qquad g(x) \neq 0$$

Example 4 - Using the Quotient Rule

Find the derivative of

$$y = \frac{5x - 2}{x^2 + 1}$$

Example 5 - Rewriting Before Differentiating

Find an equation of the tangent line to the graph of

$$f(x) = \frac{3 - (1/x)}{x + 5}$$

Example 6 - Using the Constant Multiple Rule

Original Function	Rewrite	Differentiate	Simplify
a.) $y = \dfrac{x^2 + 3x}{6}$			
b.) $y = \dfrac{5x^4}{8}$			
c.) $y = \dfrac{-3(3x - 2x^2)}{7x}$			
d.) $y = \dfrac{9}{5x^2}$			

Example 7 - Proof of the Power Rule (Negative Integer Exponents)

If n is a negative integer, there exists a positive integer k s.t. $n = -k$.

$$\frac{d}{dx}[x^n] = \frac{d}{dx}\left[\frac{1}{x^k}\right]$$

Derivatives of Trig Functions

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \tan x$$

$$\frac{d}{dx}[\csc x] = -\csc x \cot x$$

Example 8 - Differentiating Trig Functions

Find the derivative

a.) $y = x - \tan x$

b.) $y = x \sec x$

Example 9 - Different Forms of a Derivative

Differentiate both forms of

$$y = \frac{1 - \cos x}{\sin x} = \csc x - \cot x$$

First form

Second form

Higher-Order Derivatives

First Derivative	y',	$f'(x)$	$\frac{dy}{dx}$	$\frac{d}{dx}[f(x)]$	$D_x[y]$
Second Derivative	y'',	$f''(x)$	$\frac{d^2y}{dx^2}$	$\frac{d^2}{dx^2}[f(x)]$	$D_x^2[y]$
Third Derivative	y''',	$f'''(x)$	$\frac{d^3y}{dx^3}$	$\frac{d^3}{dx^3}[f(x)]$	$D_x^3[y]$
Fourth Derivative	$y^{(4)}$,	$f^{(4)}(x)$	$\frac{d^4y}{dx^4}$	$\frac{d^4}{dx^4}[f(x)]$	$D_x^4[y]$
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nth Derivative	$y^{(n)}$,	$f^{(n)}(x)$	$\frac{d^ny}{dx^n}$	$\frac{d^n}{dx^n}[f(x)]$	$D_x^n[y]$

Example 10- Finding Acceleration Due to Gravity

s(t)	Position
s'(t) = v(t)	Velocity
s''(t) = v'(t) = a(t)	Acceleration

Because the moon has no atmosphere, a falling object on the moon encounters no air resistance,. In 1971, astronomer David Scott demonstrated that a feather and a hammer fall at the same rate on the moon.
The position function for each of these falling objects is given by

$$s(t) = -.81t^2 + 2$$

where *s(t)* is the height in meters and *t* is the time in seconds.
What is the ratio of Earth's gravitational force to the moon's?