Calc

2.4 The Chain Rule

What will you learn?

- **Find the derivative of a composite function using the Chain Rule.**
- **Find the derivative of a function using the General Power Rule.**
- **Simplify the derivative of function using algebra.**
- **Find the derivative of a trig function using the Chain Rule.**

The Chain Rule

Deals with composite functions

Differentiated w/out CR

$y = x^2 + 1$
$y = \sin x$
$y = 3x + 2$
$y = x + \tan x$

Differentiated w/ CR

$y = \sqrt{x^2 + 1}$
$y = \sin 6x$
$y = (3x + 2)^5$
$y = x + \tan x^2$

Basically, the Chain Rule states that
if *y* changes *dy/du* times as fast as *u*,
and *u* changes *du/dx* times as fast as *x*,
then *y* changes *(dy/du)(du/dx)* times as fast as *x*.

In other words, the rate of change of y w/ respect to x is the product of the rate of change of y with respect to u and the rate of change of u with respect to x.

See Example 1 on Page 130

Theorem 2.10 The Chain Rule

If $y = f(u)$ is a differentiable function of u
and $u = g(x)$ is a differentiable function of x,
then $y = f(g(x))$ is a differentiable functions of x and

$$\frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dx}$$

or equivalently,

$$\frac{d}{dx}\left[\, f(g(x)) \,\right] = f\,'(g(x))\, g'(x)$$

Example 2 - Decomposition of a Composite Function

$y = f(g(x))$	$u = g(x)$	$y = f(u)$
a.) $y = \frac{1}{x + 1}$	$u = x + 1$	$y = \frac{1}{u}$
b.) $y = \sin 2x$	$u = 2x$	$y = \sin u$
c.) $y = \sqrt{3x^2 - x + 1}$	$u = 3x^2 - x + 1$	$y = \sqrt{u}$
d.) $y = \tan^2 x$	$u = \tan x$	$y = u^2$

Example 3 - Using the Chain Rule

Find ***dy/dx*** **for** $y = (x^2 + 1)^3$

The General Power Rule

If $y = [\,u(x)\,]^n$, where u is a differentiable function of x and n is a rational number,
then

$$\frac{dy}{dx} = n\,[\,u(x)\,]^{n-1}\,\frac{du}{dx}$$

or, equivalently

$$\frac{d}{dx}\,[\,u^n\,] = n\,u^{n-1}\,u'$$

Example 4 - Applying the General Power Rule

Find the derivative of

$$f(x) = (3x - 2x^2)^3$$

Example 5 - Differentiating Functions Involving Radicals

Find all points on the graph of $f(x) = \sqrt[3]{(x^2 - 1)^2}$
for which $f'(x) = 0$
and those for which $f'(x)$ ***DNE***

Example 6 - Differentiating Quotients with Constant Numerators

Differentiate

$$g(t) = \frac{-7}{(2t - 3)^2}$$

Simplifying Derivatives

Example 7 -Simplifying by Factoring Out the Least Powers

$$f(x) = x^2 \sqrt{1 - x^2}$$

Example 8 - Simplifying the Derivative of a Quotient

$$f(x) = \frac{x}{\sqrt[3]{x^2 + 4}}$$

Example 9 - Simplifying the Derivative of a Power

$$y = \left(\frac{3x - 1}{x^2 + 3} \right)^2$$

Trig Functions & the Chain Rule

$\frac{d}{dx}$ [sin u] = (cos u) u'

$\frac{d}{dx}$ [cos u] = -(sin u) u'

$\frac{d}{dx}$ [tan u] = (sec^2 u) u'

$\frac{d}{dx}$ [cot u] = - (csc^2 u) u'

$\frac{d}{dx}$ [sec u] = (sec u tan u) u'

$\frac{d}{dx}$ [csc u] = - (csc u cot u) u'

Example 10 - Applying the Chain Rule to Trig Functions

a.) *y = sin2x*

b.) *y = cos(x -1)*

c.) *y = tan 3x*

Example 11 - Parentheses & Trig Functions

a.) $y = \cos 3x^2$

b.) $y = (\cos)\, x^2$

c.) $y = \cos(3x)^2$

d.) $y = \cos^2 x$

e.) $y = \sqrt{\cos x}$

Example 12 - Repeated Application of the Chain Rule

$$f(t) = \sin^3 4t$$

Example 13 - Tangent Line of a Trig Function

Find and equation of the tangent line to the graph of

$$f(x) = 2 \sin x + \cos 2x$$

at the point (π, 1).

Then determine all the values of *x* in the interval *(0, 2π)* at which the graph of *f* has a horizontal tangent.