2.5 Implicit Differentiation

 **What will you learn?** 

- **Distinguish between functions written in the implicit form and explicit form.**
- **Use implicit differentiation to find the derivative of a function.**

Implicit & Explicit Functions

$y = 3x^2 - 5$ Explicit Form

y is written as a function of *x*

$y = \frac{1}{x}$ Implicit Form

Implicit Form	Explicit Form	Derivative
$xy = 1$	$y = \frac{1}{x} = x^{-1}$	$\frac{dy}{dx} = -x^{-2} = -\frac{1}{x^2}$

What if you have...

$$x^2 - 2y^3 + 4y = 2$$

You use *implicit differentiation*!!

Remember :

$$\frac{dy}{dx}$$

is differentiation that is taking place with respect to the variable *X* !!!

Example 1 - Differentiating *w/* respect to *x*

a.) $\frac{d}{dx}[\,x^3\,]$

b.) $\frac{d}{dx}[\,y^3\,]$

c.) $\frac{d}{dx}[\,x + 3y\,]$

d.) $\frac{d}{dx}[\,xy^2\,]$

Guidelines for Implicit Differentiation

1. **Differentiate BOTH sides of the equation *w/ respect to x***

2. **Collect all terms involving *dy/dx* on the left side and move all other terms to the right side.**

3. **Factor *dy/dx* out of the left side.**

4. **Solve for *dy/dx*.**

Example 2 - Implicit Differentiation

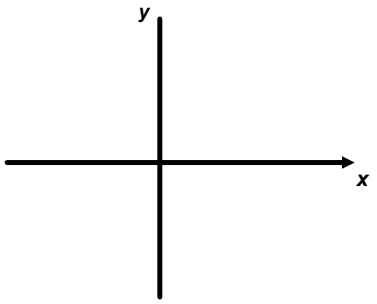
Find *dy/dx*

$$y^3 + y^2 - 5y - x^2 = -4$$

Example 3 - Representing a Graph by Differentiable Functions

If possible, represent *y* as a differentiable function of *x*

a.) $x^2 + y^2 = 0$



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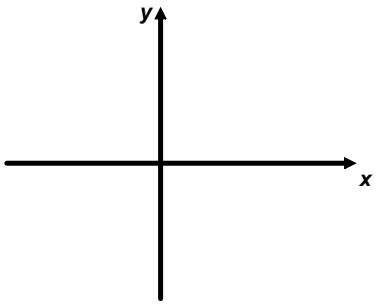
b.) $x^2 + y^2 = 1$



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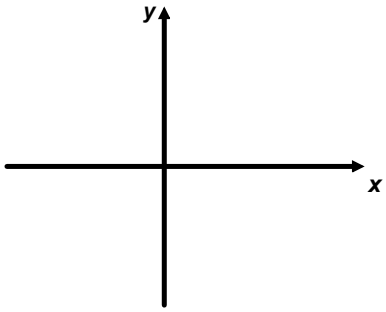
c.) $x + y^2 = 1$

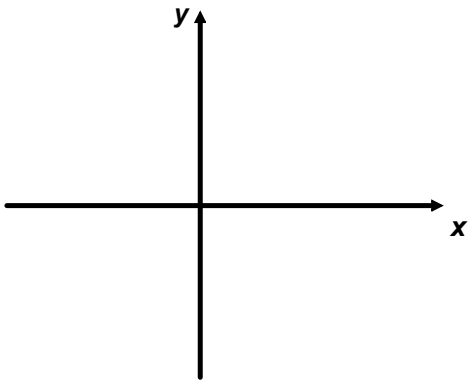


Example 4 - Finding the Slope of a Graph Implicitly

Determine the slope of the tangent line to the graph

$$x^2 + 4y^2 = 4$$

at the point $(\sqrt{2}, -1/\sqrt{2})$

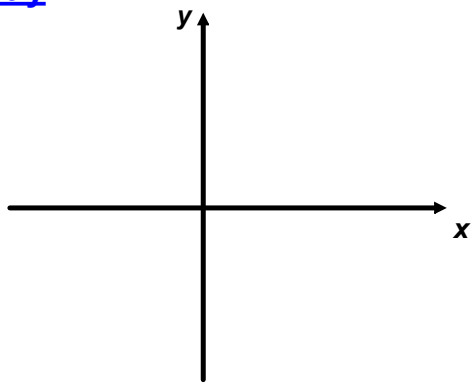


Example 5 - Finding the Slope of a Graph Implicitly

Determine the slope of the graph

$$3(x^2 + y^2)^2 = 100xy$$

at the point $(3, 1)$

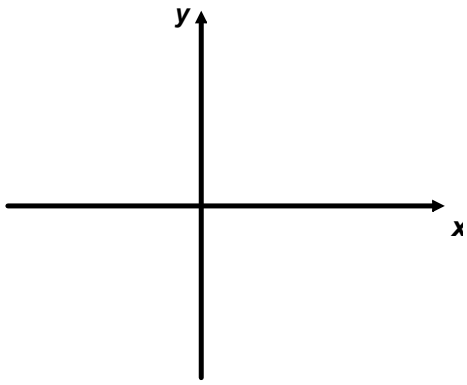


Example 6 - Determining a Differentiable Function

Find *dy/dx* implicitly for the equation

$$y = \sin x$$

Then find the largest interval of the form $-a < y < a$ on which *y* is a differentiable function of *x*.



Example 7 - Finding the Second Derivative Implicitly

Given $x^2 + y^2 = 25$

Find $\frac{d^2y}{dx^2}$

Example 8 - Finding a Tangent Line to a Graph

Find the tangent line to the graph

$$x^2 (x^2 + y^2) = y^2$$

at the point $(\sqrt{2} / 2, \sqrt{2} / 2)$