# Chapter 3 - Applications of Differentiation

## 3.1 - Extrema on an Interval

### What will you learn?

- **Understand the definition of extrema of a function on an interval.**
- **Understand the definition of relative extrema of a function on an open interval.**
- **Find the extrema on a closed interval.**

# Extrema of a Function

In calculus much effort is devoted to determining the behavior of a function on an interval *I*.

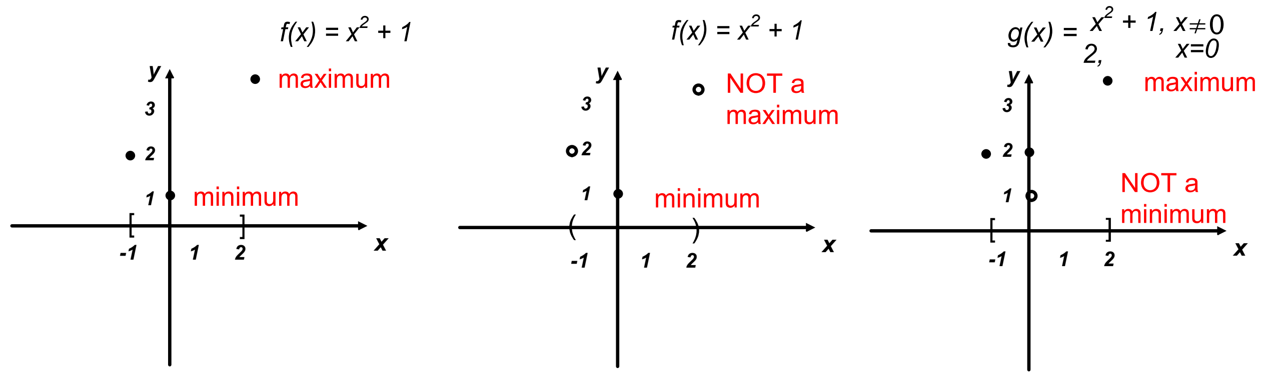
Does *f* have a maximum value on *I*?
Does it have a minimum?
Where is it increasing?
Where is it decreasing?

## Definition of Extrema

**Let *f* be defined on an interval *I* containing *c*.**

1. ***f(c)* is the minimum of *f* on *I* if $f(c) \le f(x)$ for all *x* in *I***
2. ***f(c)* is the maximum of *f* on *I* if $f(c) \ge f(x)$ for all *x* in *I***

**The minimum and maximum of a function on an interval are the extreme values, or extrema, of the function on the interval.**
**The minimum and maximum of a function on an interval are also called the absolute minimum and absolute maximum on the interval.**




f is continuous
[-1,2] is closed

f is continuous
(-1,2) is open

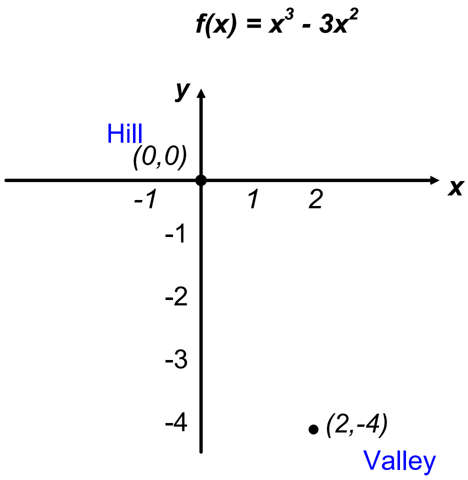
g is continuous
[-1,2} is closed

A function need not have a max or min on an interval

## The Extreme Value Theorem

**If *f* is continuous on a closed interval [a,b], then *f* has both a minimum and a maximum on the interval.**

# Relative Extrema and Critical Numbers




relative maximum at (0,0)
relative minimum at (2, -4)

Informally, you can think of a relative max as occurring on a "hill" and a relative min as occurring in a "valley'

**If the hill or valley is *smooth and rounded*, the graph has a <u>horizontal tangent line</u> at that point.**
**If the hill or valley is *sharp and peaked*, the graph represents a function that is <u>not differentiable</u> at that point.**

# Definition of Relative Extrema

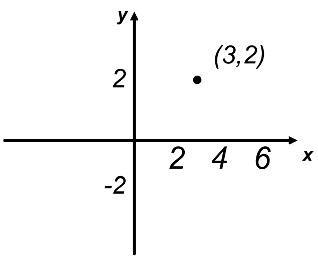
1. If there is an open interval containing $c$ on which $f(c)$ is a maximum, then $f(c)$ is called a **relative maximum** of $f$, or you can say that $f$ has a **maximum at $(c, f(c))$.**

2. If there is an open interval containing $c$ on which $f(c)$ is a minimum, then $f(c)$ is called a **relative minimum** of $f$, or you can say that $f$ has a **minimum at $(c, f(c))$.**

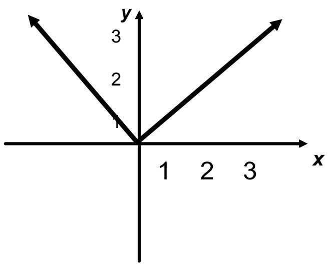
## Example 1 - The Value of the Derivative at a Relative Extrema

Find the value of the derivative at each of the relative extrema.

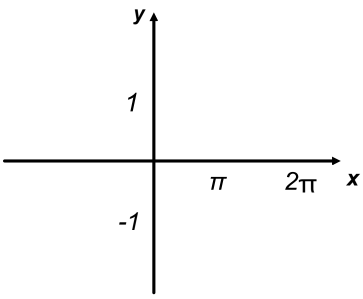
a.) $f(x) = \dfrac{9(x^2 - 3)}{x^3}$




b.) $f(x) = |x|$



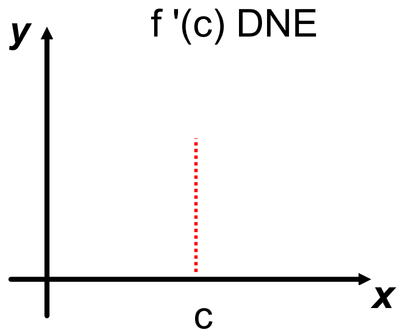

c.) $f(x) = \sin x$

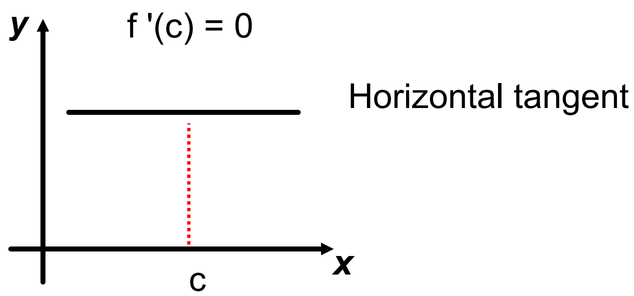


## Definition of a Critical Number

Let *f* be defined at *c*.

If *f '(c) = 0* or if *f* is not differentiable at *c*, then *c* is a critical number of *f*.




# Theorem

### Relative Extrema Occur Only at Critical Points

If *f* has a relative minimum or relative maximum at *x = c*, then *c* is a critical number of *f.*

## Finding Extrema on a Closed Interval

To find the extrema of a continuous function f on a closed interval [a,b], use the following steps

1. ***Find*** **the critical numbers of *f* on *(a,b).***

2. ***Evaluate f*** **at each critical number in *(a,b).***

3. ***Evaluate f*** **at each endpoint of *[a,b].***

4. **The *least* of these values is the minimum.**
   **The *greatest* of these values is the maximum.**

## Example 2 - Finding Extrema on a Closed Interval

**Find the extrema of**

$f(x) = 3x^4 - 4x^3$ **on the interval** $[-1, 2]$

### Example 3 - Finding the Extrema on a Closed Interval

**Find the extrema of**

$f(x) = 2x - 3x^{2/3}$ **on the interval** $[-1,3]$

## Example 4- Finding Extrema on Closed Interval

**Find the extrema of**

***$f(x) = 2 \sin x - \cos 2x$*** **on the interval** ***$[1, 2\pi]$***