3.2 Rolle's Theorem and the Mean Value Theorem

☺ **What will you learn?** ☺

- **Understand and use Rolle's Theorem**
- **Understand and use the Mean Value Theorem**

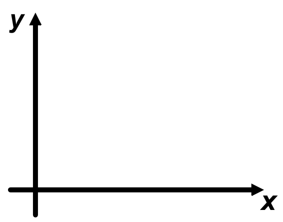
Rolle's Theorem

Let *f* be continuous on the closed interval [a,b] and differentiable on the open interval (a,b). If

$$f(a) = f(b)$$

then there is at least one number c in (a,b) s.t. $f'(c) = 0$

From Rolle's Theorem, you can see that if a function *f* is continuous on [a,b] and differentiable on *(a,b)*, and if *f(a)* = *f(b)*, there must be at least one *x* - value between *a* and *b* at which the graph of *f* has a horizontal tangent.

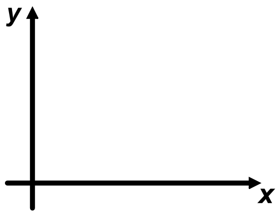


Example 1 - Illustrating Rolle's Theorem

Find the two x-intercepts of

$$f(x) = x^2 - 3x + 2$$

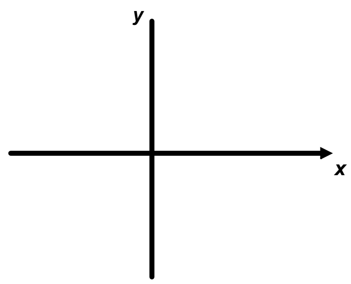
and show that $f'(x) = 0$ at some point between the two x- intercepts



Example 2 - Illustrating Rolle's Theorem

Let $f(x) = x^4 - 2x^2$

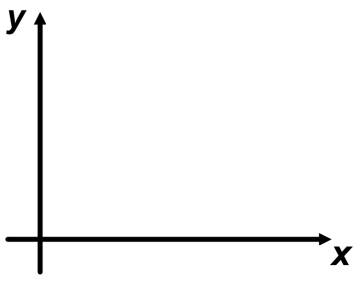
Find all the values of c **in the interval** $(-2, 2)$ **s.t.** $f'(c) = 0$



The Mean Value Theorem

If *f* is continuous on the closed interval [a,b,] and differentiable on the open interval (a,b), then there exists a number *c* in (a,b) s.t.

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

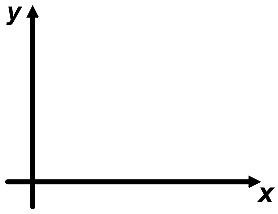


Example 3 - Finding a Tangent Line

Given $f(x) = 5 - (4/x)$

Find all the values of c **in the open interval** $(1,4)$ **s.t.**

$$f'(c) = \frac{f(4) - f(1)}{4 - 1}$$



Example 4 - Finding an Instantaneous Rate of Change

Two stationary patrol cars equipped with radar are 5 miles apart on a highway. As a truck passes the first patrol car, its speed is clocked at 55 mph. Four minutes later, when the truck passes the second patrol car, its speed is clocked at 50mph. Prove that the truck must have exceeded the speed limit of 55 mph at some time during the 4 minutes