3.3 Increasing and Decreasing Functions and the First Derivative Test

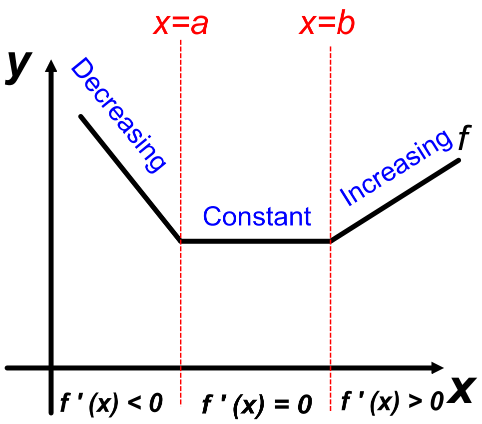
What will you learn?

- **Determine intervals on which a function is increasing or decreasing**
- **Apply the first derivative test to find relative extrema of a function**

Increasing & Decreasing Functions

A function f is ***increasing*** on an interval if for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) < f(x_2)$.

A function f is ***decreasing*** on an interval if for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.



Theorem 3.5 - Test for Increasing & Decreasing Functions

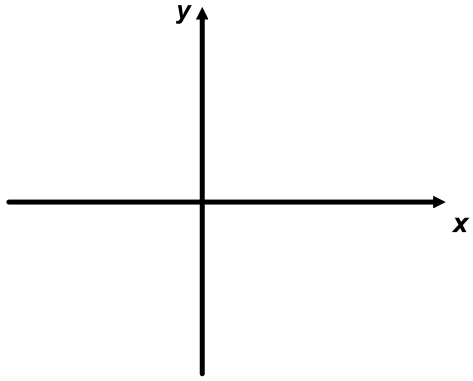
Let f be a function that is *continuous* on the closed interval $[a,b]$ and *differentiable* on the open interval (a,b)

1. If $f'(x) > 0$ for all x in (a,b), then f is *increasing* on $[a,b]$.
2. If $f'(x) < 0$ for all x in (a,b), then f is *decreasing* on $[a,b]$.
3. If $f'(x) = 0$ for all x in (a,b), then f is *constant* on $[a,b]$.

Example 1 - Intervals on which *f* is Increasing or Decreasing

Find the open intervals on which $f(x) = x^3 - \frac{3}{2}x^2$

is increasing or decreasing



Guidelines for Finding Intervals on Which a Function is Increasing or Decreasing

Let *f* be continuous of the interval *(a,b)*.
To find the open intervals on which *f* is increasing or decreasing, use the following steps:

1. **Locate the *critical numbers* of *f* in *(a,b)*, and use these numbers to determine the test intervals.**

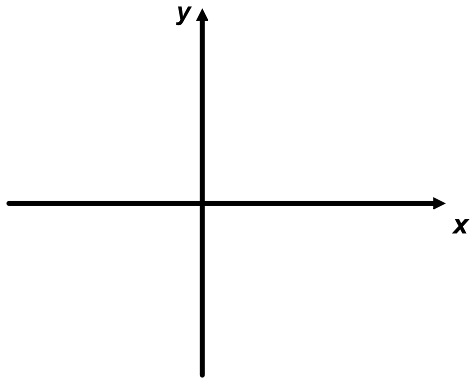
2. **Determine the sign of *f '(x)* at one test value in each of the intervals.**

3. **Use Theorem 3.5 to determine whether *f* is increasing or decreasing on each interval.**

These guidelines are also valid if the interval (a,b) is replaced by an interval of the form *(-∞, b), (a, ∞) or (-∞, ∞)*

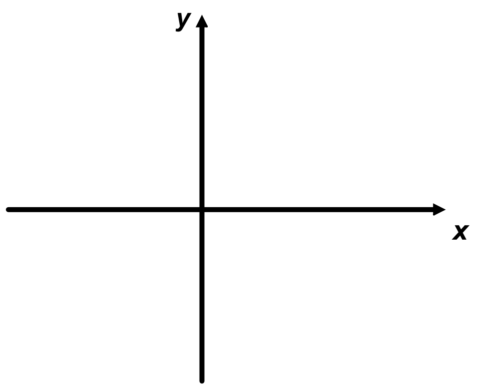
A function is strictly monotonic on an interval if it is either increasing on the entire interval or decreasing on the entire interval.

eg.) $f(x) = x^3$



The First Derivative Test

Given $f(x) = x^3 - \frac{3}{2}x^2$



It is not difficult to locate the relative extrema of this function

Relative Maximum at (0,0) because f is increasing immediately to the left of x = 0 and decreasing immediately to the right of x = 0.

Relative Minimum at (1, - 1/2) because f is decreasing immediately to the left of x=1 and increasing immediately to the right of x = 1

Theorem 3.5 The First Derivative Test

Let *c* be a critical number of a function *f* that is continuous on an open interval *I* containing *c*.

If *f* is differentiable on the interval, except possible a*t c*, then *f(c)* can be classified as follows:

1. **If *f '(x)* changes from negative to positive at *c*, then f has a *relative minimum* at *(c, f(c)).***

2. **If *f '(x)* changes from positive to negative at *c*, then f has a *relative maximum* at *(c, f(c)).***

3. **If *f* is positive on both sides of c or negative on both sides of *c*, then *f(c)* is *neither* a relative minimum nor a relative maximum.**

Example 2 - Applying the First Derivative Test

Find the relative extrema of the function

$$f(x) = \frac{1}{2}x - \sin x \qquad \text{on the interval } (0, 2\pi)$$

Example 3 - Applying the First Derivative Test

Find the relative extrema of

$$f(x) = (x^2 - 4)^{2/3}$$

Be sure you consider
the <u>domain</u>
of the function
when using the 1st Derivative Test

$$f(x) = \frac{x^4 + 1}{x^2}$$

is not defined when x = 0

This x-value MUST be used as a
CRITICAL NUMBER
to determine the test intervals!

Example 4 - Applying the First Derivative Test

Find the relative extrema of

$$f(x) = \frac{x^4 + 1}{x^2}$$

Example 5 - The Path of a Projectile

Neglecting air resistance, the path of a projectile that is propelled at an angle θ is

$$y = \frac{g \sec^2 \theta}{2v_0^2} + (\tan \theta)\, x + h \qquad 0 \le \theta \le \frac{\pi}{2}$$

y = height
x = horizontal distance
g = acceleration due to gravity
v_0= initial velocity
h = initial height

Let g = -32 feet per second,
v_0= 24 feet per second
h = 9 feet

What value of θ will produce a maximum horizontal distance?