Chapter 4 - Integration

4.1 Antiderivatives and Indefinite Integration

What will you learn?

- **Write the general solution of a differential equation.**
- **Use indefinite integral notation to find antiderivatives**
- **Use basic integration rules to find antiderivatives**
- **Find a particular solution of a differential equation**

Antiderivatives

The opposite of differentiation!

Given the derivative...can you find the original
function from which the derivative came?

Find a function *F* whose derivative is $f(x) = 3x^2$

Definition of Antiderivative

A function *F* is an antiderivative of *f* on an interval *I* if

$F'(x) = f(x)$ for all x in I

Note : *F* is an antiderivative not the antiderivative.....why?

Theorem 4.1 - Representation of Antiderivatives

If F is an antiderivative of f on an interval I, then G is an antiderivative of f on the interval I iff G is of the form $G(x) = F(x) + C$, for all x in I where C is a constant.

Example 1 - Solving a Differential Equation

Find the general solution of the differential equation $y' = 2$

Notation for Antiderivatives

When solving a differential equation of the form

$$\frac{dy}{dx} = f(x)$$

it is convenient to write it in the equivalent differential form

$$dy = f(x)\,dx$$

The operation finding all solutions of this equation is called antidifferentiation (or indefinite integral)

Variable of integration

$$y = \int f(x)\,dx = F(x) + C$$

Integrand

Constant of Integration

Basic Integration Rules

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Example 2 - Applying the Basic Integration Rules

Describe the antiderivatives of *3x*

Example 3 - Rewriting Before Integrating

Original Integral	Rewrite	Integrate	Simplify
a.) $\int \frac{1}{x^3}\, dx$			
b.) $\int \sqrt{x}\, dx$			
c.) $\int 2 \sin x\, dx$			

Example 4 - Integrating Polynomial Functions

a.) $\int dx$

b.) $\int (x + 2)\, dx$

c.) $\int (3x^4 - 5x^2 + x)\, dx$

Example 5 - Rewriting Before Integrating

$$\int \frac{x+1}{\sqrt{x}}\,dx$$

Example 6 - Rewriting Before Integrating

$$\int \frac{\sin x}{\cos^2 x}\, dx$$

Initial Conditions and Particular Solutions

You already know that
$y = \int f(x)\,dx$
has many solutions

Each differing by a constant

$\therefore$ Any 2 antiderivatives of f are vertical translations or each other

We usually want a PARTICULAR solution

To do this we need only to know the value of $y = F(x)$ for only one value of x

We call this the **INITIAL CONDITION**

Example 7 - Finding a Particular Solution

Find the general solution of

$$F'(x) = \frac{1}{x^2} \qquad x > 0$$

and find the particular solution that satisfies the initial condition $F(1) = 0$

Example 8 - Solving a Vertical Motion Problem

A ball is thrown upward with an initial velocity of 64 feet per second from an initial height of 80 feet

a.) Find the position function giving the height *s* as a function of the time *t*

b.) When does the ball hit the ground?