Chapter 1 - Limits & Their Properties

☆ **The limit process is a fundamental concept of calculus.** ☆

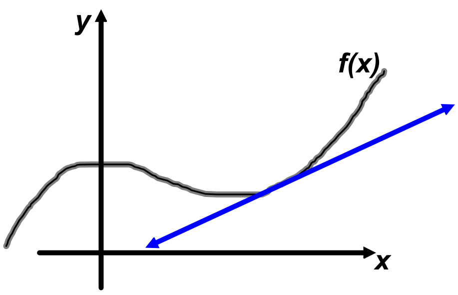
☺ You will learn how to find limits ☺

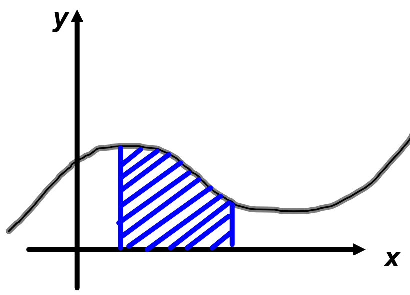
- analytically
- graphically
- numerically

1.1 A Preview of Calculus

What will you learn???

- Understand what calculus is and how it compares with precal
- Understand that the *tangent line problem* is basic to calculus
- Understand that the *area problem* is also basic to calculus





So....What is Calculus anyway???

Calculus is the mathematics of CHANGE

- **velocities**
- **accelerations**

Calculus is the mathematics of

- **tangent lines**
- **slopes, areas, volumes**
- **arc lengths**
- **centroids**
- **curvatures**
- **other scientific,engineering & economic situations**

What's the difference between Precal and Calc????

Precal is more static

Calc is more dynamic

Examples:

An object traveling at a *constant velocity* → *precal*
To analyze the velocity of an *accelerating object* → *calc*

The *slope of a line* → *precal*
The *slope of a curve* → *calc*

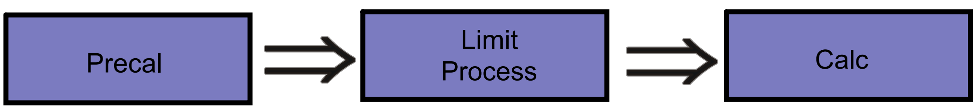
The *tangent line to a circle* → *precal*
The *tangent line to a general graph* → *calc*

The *area of a rectangle* → *precal*
The *area under a general curve* → *calc*

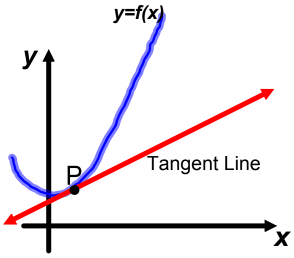
See p. 43 & 44 for visual examples

What is Calculus?

"limit machine"



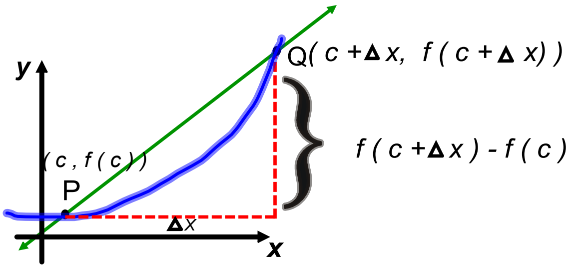
The Tangent Line

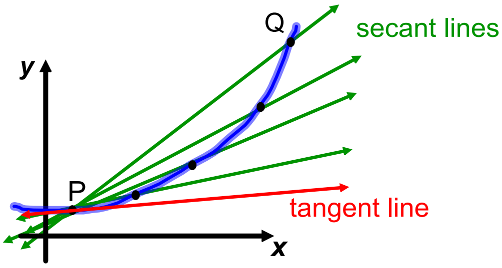


The problem of finding the **tangent line** at a point P is equivalent to finding the ***slope*** of the tangent line at point P (except for vertical tangent lines)

Using **secant lines**

$$m_{sec} = \frac{f(c+\Delta x) - f(c)}{c+\Delta x - c} = \frac{f(c+\Delta x) - f(c)}{\Delta x}$$





As a point Q approaches P, the slope of the secant line approaches the slope of the tangent line

The slope of the tangent line is said to be the ***LIMIT*** of the slope of the secant line

Exploration

The following points lie on the graph of $f(x) = x^2$

$Q_1\ (1.5, f(1.5))$

$Q_2\ (1.1, f(1.1))$

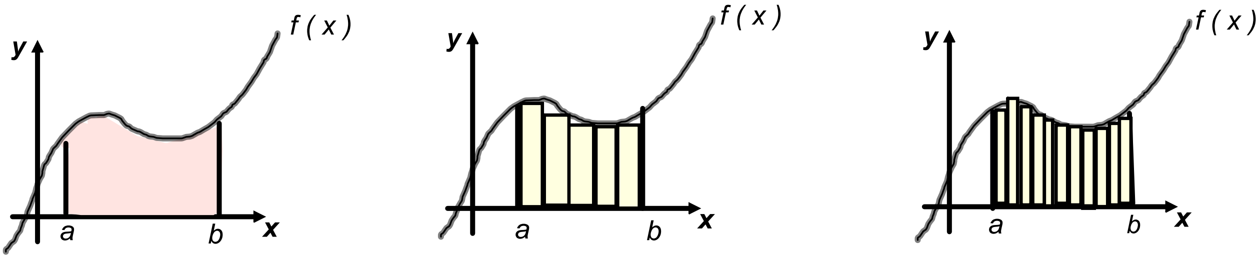
$Q_3\ (1.01, f(1.01))$

$Q_4\ (1.001, f(1.001))$

$Q_5\ (1.001, f(1.001))$

Each successive point gets closer and closer to the poin $P\ (1,1)$.
Find the slope of each secant line through Q_1 and P, Q_2 and P ...
Graph the secant lines on a graphing calculator.
Use the results to estimate the slope of the tangent line to the graph of f at P.

The Area Problem



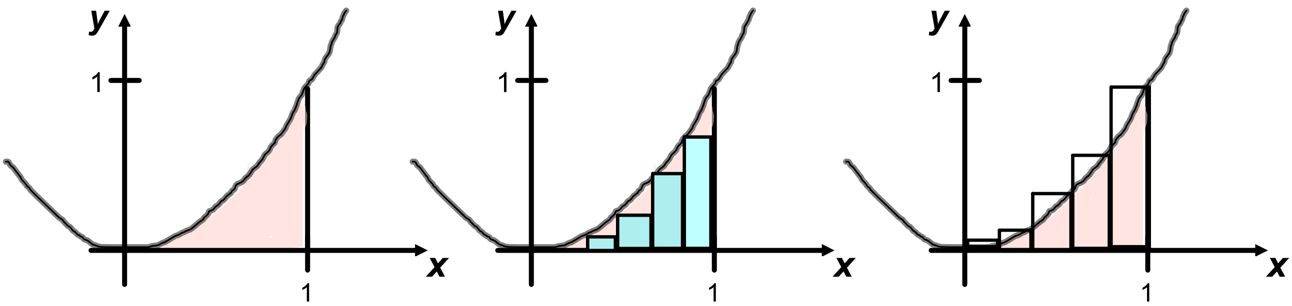
Consider the region bounded by the graph of *f (x)* , the *x*-axis, and the vertical lines $x = a$ and $x = b$.

As you increase the number of rectangles, the approximation tends ot become better and better because the amount of area missed by the rectangles decreases.

Your goal is to determine the limit of the sum of the areas of the rectangles as the number of rectangles increases without bound.

Exploration

Consider the region bounded by the graphs of f (x) = x^2 , *y = 0* and *x = 1.*



The area of the region can be approximated by two sets of rectangles -
one set inscribed within the region and the other set circumscribed over the region.

Find the sum of the areas of each set of rectangles.

Then use your results to approximate the area of the region.