1.2 Finding the Limits Graphically & Numerically

☺ What will you learn? ☺

- **Estimate a limit using a *numerical* or *graphical* approach.**
- **Learn different ways that a *limit can fail to exist*.**
- **Study and use *formal definition of limit*.**

An Intro to Limits

Given

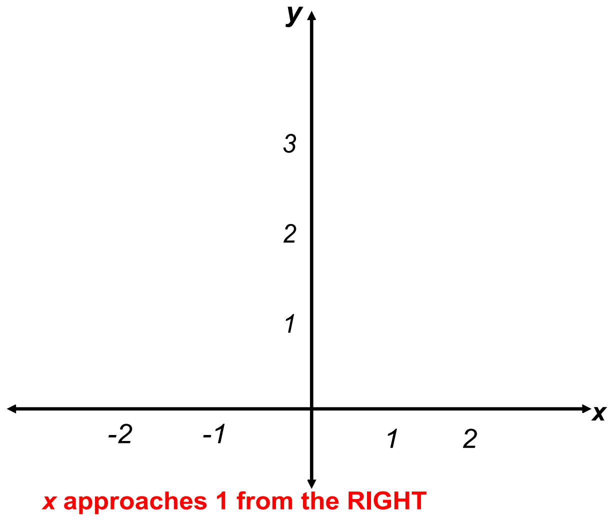
$$f(x) = \frac{x^3 - 1}{x - 1}, \quad x \neq 1$$

Sketch a graph of this function

What do you KNOW about this function
(remember Precal???) :)

How do you factor a cubic???

What's happening at $x = 1$?



x **approaches 1 from the LEFT** → ← *x* **approaches 1 from the RIGHT**

x	0.75	0.9	0.99	0.999	1	1.001	1.01	1.1	1.25
f (x)									

→ ←

***f (x)* approaches** __________ ***f (x)* approaches** __________

The graph of *f* is a parabola that has a gap at the point *(1, 3)* .

Although x cannot equal 1, you can move arbitrarily close to 1, and as a result f(x) moves arbitrarily close to 3

$$\lim_{x \to 1} f(x) = 3$$

<u>Limit</u>

(Informal description of a limit)

$$\lim_{x \to c} f(x) = L$$

If *f (x)* becomes arbitrarily close to a single number *L*
as *x* approaches *c* from either side,
the <u>LIMIT</u> of *f (x)* as *x* approaches *c*, is *L.*

Exploration

Given

$$\lim_{x \to 2} \frac{x^2 - 3x + 2}{x - 2}$$

1. Estimate the limit *numerically*

   Using a table of values

2. Estimate the limit *graphically*

   Using your graphing calculator

Example 1 - Estimating a Limit Numerically

Evaluate the function $f(x) = \dfrac{x}{\sqrt{x+1} - 1}$

at several points near $x = 0$ and use the results to estimate the limit

$$\lim_{x \to 0} \frac{x}{\sqrt{x+1} - 1}$$

x approaches 0 from the LEFT → ← x approaches 0 from the RIGHT

x	-0.01	-0.001	-0.0001	0	0.0001	0.0001	0.01
$f(x)$							

f (x) approaches________ → ← *f (x)* approaches________

NOTE : The function is UNDEFINED at $x = 0$,
yet *f (x)* appears to approaching a limit as x approaches 0

☆ **IMPORTANT** ☆

The ***existence or nonexistence*** **of** $f(x)$ **at** $x = c$

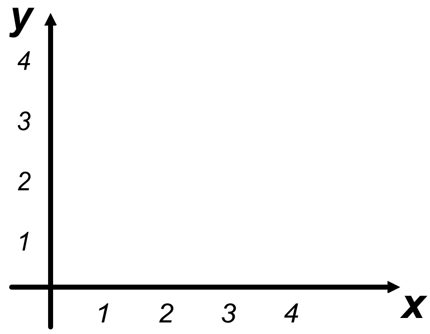
has no bearing on the

existence of the limit of $f(x)$ ***as*** x ***approaches*** c **!**

Example 2 - Finding a Limit

Find the limit of *f (x)* as x approaches *2* where *f* is defined as:

$$f(x) = \begin{cases} 1, & x \neq 2 \\ 0, & x = 2 \end{cases}$$



Three - pronged approach to problem solving

(you need to know all of these for the AP Exam!)

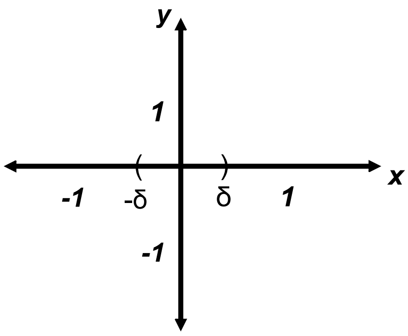
1. Numerical Approach →
2. Graphical Approach →
3. Analytic Approach →

Limits That Fail to Exist

Example 3 - Behavior that Differs from the Right & Left

Show that the limit does not exist.

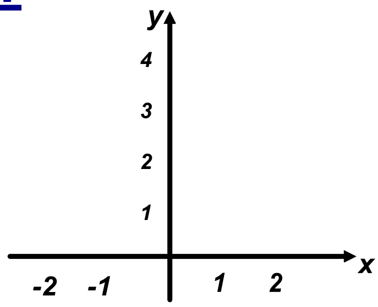
$$\lim_{x \to 0} \frac{|x|}{x}$$



δ ⟶ Lowercase *delta*

Example 4 - Unbounded Behavior

$$\lim_{x \to 0} \frac{1}{x^2}$$



Example 5 - Oscillating Behavior

$$\lim_{x \to 0} \sin \frac{1}{x}$$

x	$\frac{2}{\pi}$	$\frac{2}{3\pi}$	$\frac{2}{5\pi}$	$\frac{2}{7\pi}$	$\frac{2}{9\pi}$	$\frac{2}{11\pi}$	$x \to 0$
$\sin(1/x)$							

Be careful when examining *sin (1/x)* with graphing calc. - may not get a clear picture

Limits that DNE

1. $f(x)$ approaches a different number from the right of c than in approaches form the left of c

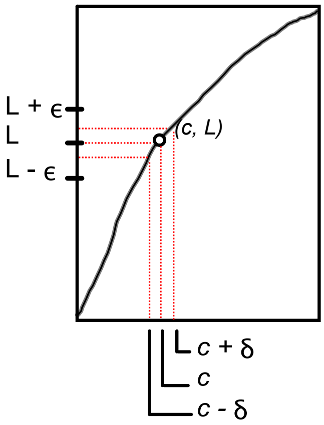
2. $f(x)$ increases or decreases without bound as $x \longrightarrow c$

3. $f(x)$ oscillates between 2 fixed valuess as $x \longrightarrow c$

A Formal Definition of Limit

If *f (x)* becomes arbitrarily close to a single number *L* as *x* approaches *c* from *either side*, then the limit of *f (x)* as *x* approaches *c* is *L*, written as

$$\lim_{x \to 0} f(x) = L$$



ϵ –δ definition of limit

Definition of Limit

Let *f* be a function defined on an open interval containing *c* (except possibly at *c*) and *L* be a real number. The statement

$$\lim_{x \to c} f(x) = L$$

means that for each $\epsilon > 0$ there exists a $\delta > 0$ s.t. if

$0 < |x - c| < \delta$ ***then*** $|f(x) - L| < \epsilon$

Some functions do not have limits as x ⟶c, but those that do cannot have two different limits as x⟶c.

If the limit of a function exists, it is unique.