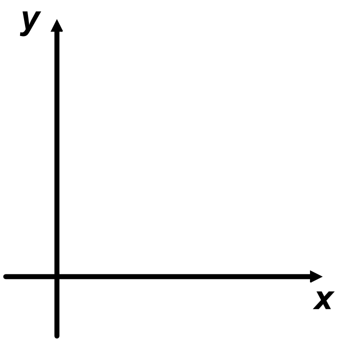
Calc

1.4 Continuity & One-Sided Limits

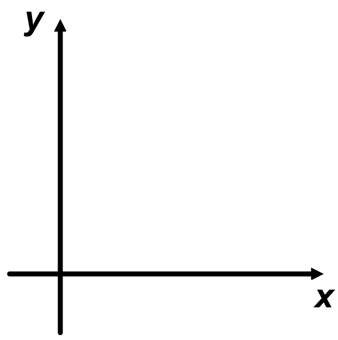
What will you learn?

- Determine continuity at a point and continuity on an open interval
- Determine one-sided limits & continuity on a closed interval
- Use properties of continuity
- Understand & use Intermediate Value Theorem

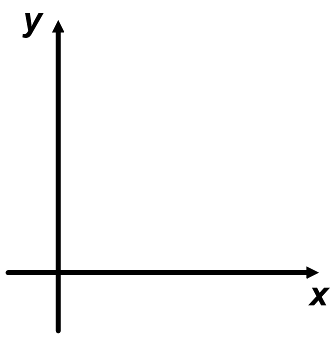
Continuity at a Point and on an Open Interval



f(c) in NOT DEFINED



$\lim_{x \to c} f(x) \longrightarrow$ DNE



$\lim_{x \to c} f(x) \neq f(c)$

Definition of Continuity

Continuity at a Point

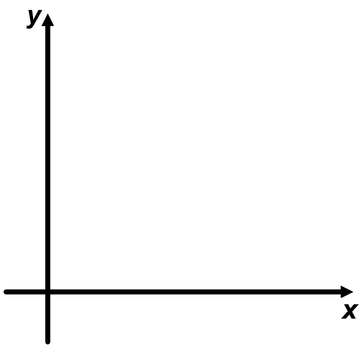
A function *f* is continuous at *c* if the following 3 conditions are met.

1. **f(c) is defined**
2. **lim *f(x)* exists**
   x c
3. **lim *f(x)* = *f(c)***
   x c

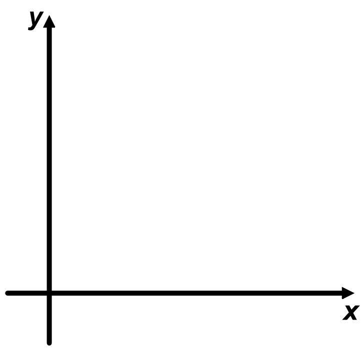
Continuity on an Open Interval

**A function is continuous on an open interval (a, b)
if it continuous at each point in the interval.
A function that is continuous on the entire real line ($-\infty$, ∞)
is everywhere continuous.**

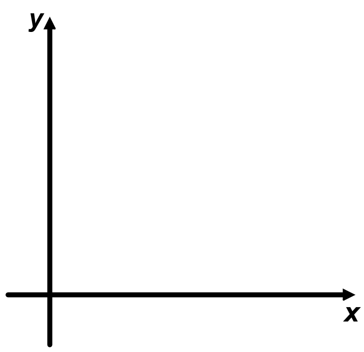
Discontinuities



Removable



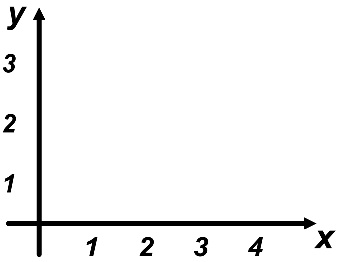
Nonremovable



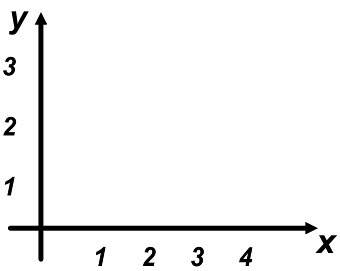
Removable

Example 1 - Continuity of a Function

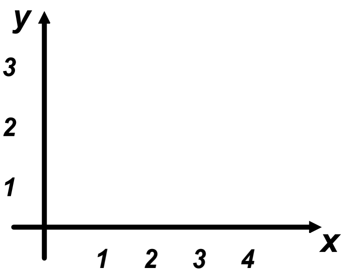
a.) $f(x) = \frac{1}{x}$



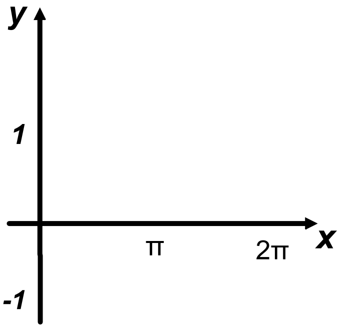
b.) $g(x) = \frac{x^2 - 1}{x - 1}$



c.) $h(x) = \begin{cases} x + 1, & x \leq 0 \\ x^2 + 1, & x > 0 \end{cases}$



d.) $y = \sin x$



One-Sided Limits & Continuity on a Closed Interval

Limit from the Right ⟶ *x* approaches *c* from values greater than *c*

$$\lim_{x \to c^+} f(x) = L$$

Limit from the left ⟶ *x* approaches *c* from values less than *c*

$$\lim_{x \to c^-} f(x) = L$$

Example 2 - A One-Sided Limit

Find the limit of $f(x) = \sqrt{4 - x^2}$
as x approaches -2 from the right

Example 3 - The Greatest Integer Function

Find the limit of the greatest integer function *f(x) = [[x]]* as *x* approaches *0* from the left and right

Theorem 1.10 The Existence of a Limit

Let f be a function and let c and L be real numbers.
The limit of $f(x)$ as x approaches c is L iff

$$\lim_{x \to c^-} f(x) = L \quad \text{and} \quad \lim_{x \to c^+} f(x) = L$$

Definition of Continuity on a Closed Interval

A function f is continuous on the closed interval $[a, b]$ if it is continuous on the open interval (a, b) and

$$\lim_{x \to a^+} f(x) = f(a) \quad \text{and} \quad \lim_{x \to b^-} f(x) = f(b)$$

The function is continuous from the right at a and continuous from the left at b

Example 4 - Continuity on a Closed Interval

Discuss the continuity of $f(x) = \sqrt{1 - x^2}$

Example 5 - Charles's Law & Absolute Zero

Theorem 1.11 Properties of Continuity

If *b* is a real number and *f* and *g* are continuous at $x = c$, then the functions are also continuous at *c.*

1. **Scalar Multiple :** bf
2. **Sum & Difference :** $f \pm g$
3. **Product:** fg
4. **Quotient:** $\frac{f}{g}$ if $g(c) \neq 0$

The following types of functions are continuous at every point in their domain:

1. Polyomial
2. Rational
3. Radical
4. Trig

Example 6 - Applying Properties of Continuity

By Theorem 1.11 - all of the following functions are *continuous at every point in its domain*

$$f(x) = x + \sin x$$

$$f(x) = 3 \tan x$$

$$f(x) = \frac{x^2 + 1}{\cos x}$$

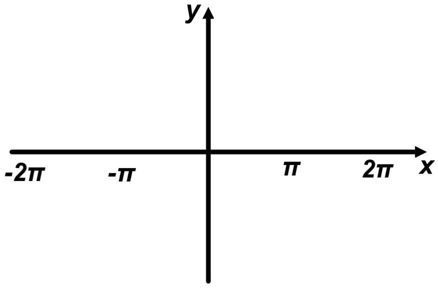
Theorem 1.12 - Continuity of a Composite Function

If g is continuous at c and f is continuous at $g(c)$,
then the composite function given by $(f \circ g)(x)$ is continuous at c.

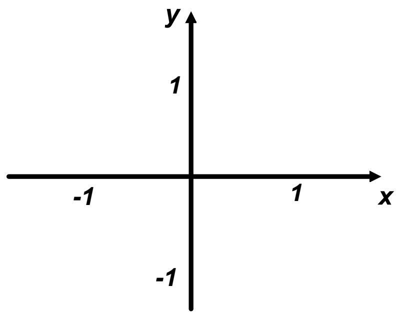
Example 7 - Testing for Continuity

Describe the intervals on which each function is continuous

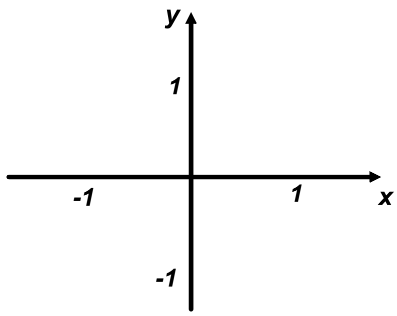
a.) $f(x) = \tan x$



b.) $g(x) = \begin{cases} \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$



c.) $h(x) = \begin{cases} x \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

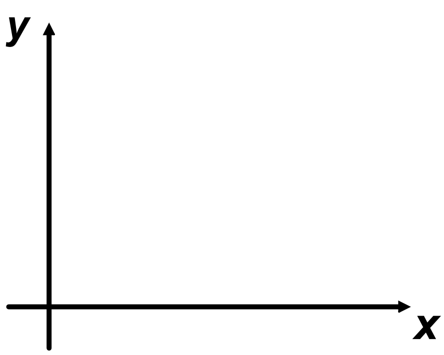


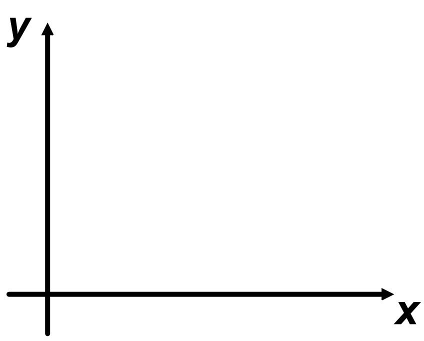
Theorem 1.13 The Intermediate Value Theorem

**If *f* is continuous on the closed interval *[a, b]* and *k* is any number between *f(a)* and *f(b)*,
then there is at least one number *c* in *[a, b]* s.t.**

$$f(c) = k$$

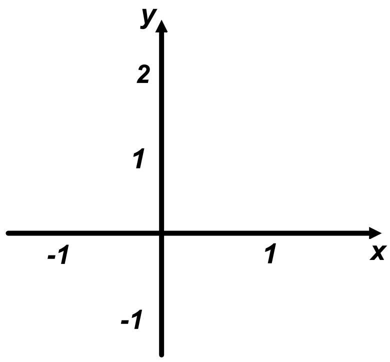
This is an existence theorem - it tells you that at least one **c** exists, but it does not give a method for finding *c*





Example 8- An Application of the IVT

Use the IVT to show that the polynomial function $f(x) = x^3 + 2x - 1$ has a zero in the interval [0, 1]



Bisection Method