1.5 Infinite Limits

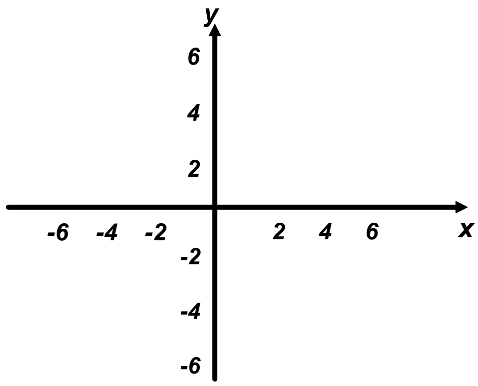
What will you learn??

- Determine infinite limits from the left & from the right
- Find & sketch the VA's of the graph of a function

Infinite Limits

Let *f* be the function given by

$$f(x) = \frac{3}{x - 2}$$



$\lim_{x \to 2^-} f(x) =$

$\lim_{x \to 2^+} f(x) =$

x	1.5	1.9	1.99	1.999	2	2.001	2.01	2.1	2.5
f(x)					?				

Definition of Infinite Limits

Let f be a function that is defined at every real number in some open interval containing c (except possibly c itself). The statement

$$\lim_{x \to c} f(x) = \infty$$

means that for each $M > 0$ there exists a $\delta > 0$ s.t. $f(x) > M$ whenever $0 < x - c < \delta$. Similarly, the statement

$$\lim_{x \to c} f(x) = -\infty$$

means that for each $N < 0$ there exists a $\delta > 0$ s.t. $f(x) < N$ whenever $0 < |x - c| < \delta$.

To define infinite limit from the left, replace $0 < |x - c| < \delta$ by $c - \delta < x < c$

To define infinite limit from the right, replace $0 < |x - c| < \delta$ by $c < x < c + b$

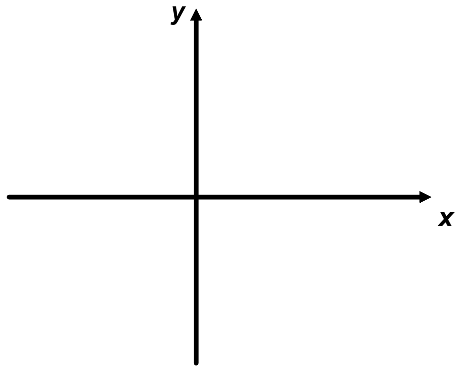
Be sure you see that the equal sign in the statement lim $f(x) = \infty$ DOES NOT MEAN that the limit exists!

On the contrary, it tells you how the limit FAILS TO EXIST by denoting the unbounded behavior of $f(x)$ as x approaches c

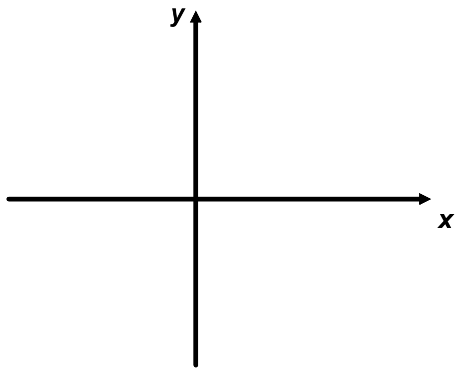
Example 1- Determining Infinite Limits from a Graph

Use the following graphs to determine the limit of each function as *x* approaches *1* from the left and from the right

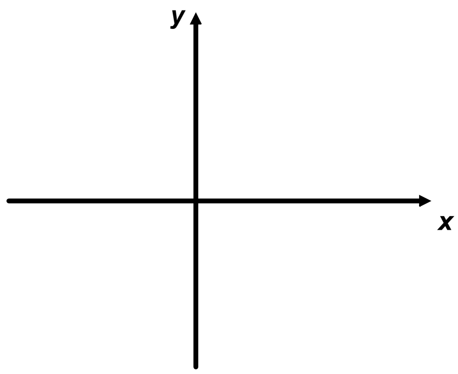
a.) $f(x) = \dfrac{1}{x - 1}$



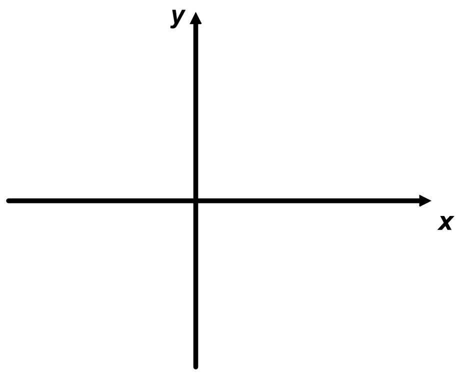
b.) $f(x)= \dfrac{1}{(x - 1)^2}$



c.) $f(x)= \dfrac{-1}{x - 1}$



d.) $f(x)= \dfrac{- 1}{(x - 1)^2}$



Vertical Asymptotes

If $f(x)$ approaches $\pm\infty$ as x approaches c from the right or left, then the line $x = c$ is a ***vertical asymptote*** of the graph of f.

Theorem 1.14 - Vertical Asymptotes

Let f and g be continuous on an open interval containing c.
If $f(c) \neq 0$, $g(c) = 0$, and there exists an open interval containing c s.t. $g(x) \neq 0$ for all $x \neq c$ in the interval, then the graph of the function given by

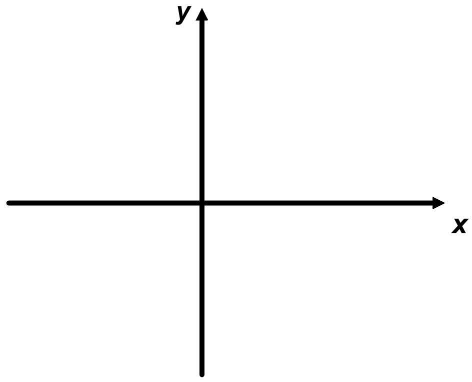
$$h(x) = \frac{f(x)}{g(x)}$$

has a vertical asymptote at $x = c$.

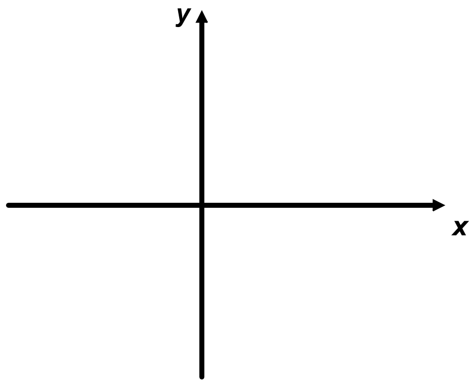
Example 2 - Finding Vertical Asymptotes

Determine all the vertical asymptotes of the graphs of each function

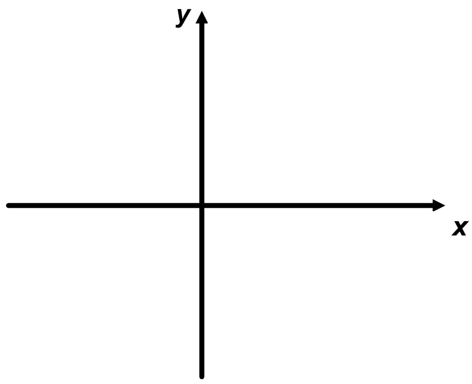
a.) $f(x) = \dfrac{1}{2(x + 1)}$



b.) $f(x) = \dfrac{x^2 + 1}{x^2 - 1}$



c.) $f(x) = \cot x$



Example 3 - A Rational Function with Common Factors

Determine all the V.A.'s of the graph of

$$f(x) = \frac{x^2 + 2x - 8}{x^2 - 4}$$

Example 4 - Determining Infinite Limits

Find each limit

$$\lim_{x \to 1^-} \frac{x^2 - 3x}{x - 1} \qquad \text{and} \qquad \lim_{x \to 1^+} \frac{x^2 - 3x}{x - 1}$$

Theorem 1.15 - Properties of Infinite Limits

Let c and L be real numbers and let f and g be functions s.t.

$$\lim_{x \to c} f(x) = \infty \quad \text{and} \quad \lim_{x \to c} g(x) = L$$

1. **Sum or Difference** $\lim_{x \to c} [\, f(x) \pm g(x)] = \infty$

2. **Product** $\lim_{x \to c} [\, f(x)g(x)] = \infty, \quad L > 0$

   $\lim_{x \to c} [\, f(x)g(x)] = -\infty, \quad L < 0$

3. **Quotient** $\lim_{x \to c} \dfrac{g(x)}{f(x)} = 0$

Similar properties hold for on-sided limits and for functions for which the limit of f(x) as x approaches c is $-\infty$

Example 5 - Determining Limits

a. Because $\lim_{x \to 0} 1 = 1$ and $\lim_{x \to 0} \frac{1}{x^2} = \infty$, you can write

b. Because $\lim_{x \to 1^-} (x^2 + 1) = 2$ and $\lim_{x \to 1^-} \cot \pi x = -\infty$, you can write

c. Because $\lim_{x \to 0^+} 3 = 3$ and $\lim_{x \to 0^+} \cot x = \infty$, you can write