Calc

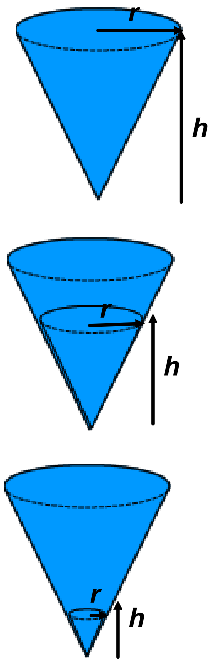
2.6 Related Rates

What will you learn?

- **Find a related rate.**
- **Use related rates to solve real-life problems.**

Finding Related Rates

Finding the rates of change of two or more related variables that are changing with respect to time



EG) When water is drained out of a conical tank, the volume V, the radius r, and the height h, are all functions of time t

$$V = \frac{\pi}{3} r^2 h$$

you can *differentiate implicitly* with respect to t, to obtain the related-rate

$$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{\pi}{3} r^2 h\right)$$

Example 1 - Two Rates That Are Related

Suppose x and y are both differentiable functions of t and are related by the equation

$$y = x^2 + 3$$

Find $\frac{dy}{dt}$ when $x = 1$, given that $\frac{dx}{dt} = 2$ when $x = 1$.

Example 2 - Ripples in a Pond

A pebble is dropped into a calm pond, causing ripples in the form of concentric circles.

The radius r of the outer ripple is increasing at a constant rate of 1 foot per second.

When the radius is 4 feet, at what rate is the total area A of the disturbed water changing?

Guidelines for Solving Related-Rates Problems

1. Determine: What is *given*?
   What do you*have to find*?
   Make a sketch.

2. Write and equation

3. Use Chain Rule to implicitly differentiate BOTH sides of the equation
   with *respect to time t.*

4. Substitute all known values for the variables and their rates of change into the equation.
   Solve for the required rate of change.

Examples of mathematical models involving rates of change

Verbal Statements	Mathematical Model
The velocity of a car after traveling for 1 hour is 50 miles per hour.	x = distance traveled $\frac{dx}{dt} = 50$ when $t = 1$
Water is being pumped into a swimming pool at a rate of 10 cubic meters per hour.	V = volume of water in pool $\frac{dV}{dt} = 10$ m³/hr
A gear is revolving at a rate of 25 revolutions per minute. (1 revolution = 2π radians)	θ = angle of revolution $\frac{d\theta}{dt} = 25(2\pi)$ rad/min

Example 3 - An Inflating Balloon

Air is being pumped into a spherical shaped balloon at a rate of 4.5 cubic feet per minute.

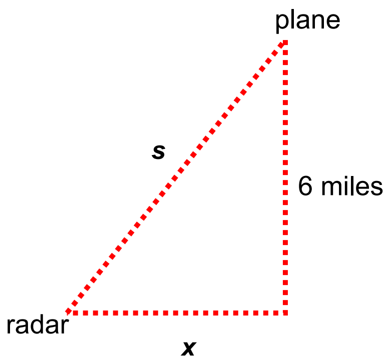
Find the rate of change of the radius when the radius is 2 feet.

Volume of a sphere $V = \frac{4}{3}\pi r^3$

Example 4 - The Speed of an Airplane Tracked by Radar

An airplane is flying on a flight path that will take it directly over a radar tracking station.

If *s* is decreasing at a rate of *400 mph* when *s = 10 miles*, what is the speed of the plane?



Example 5 - A Changing Elevation

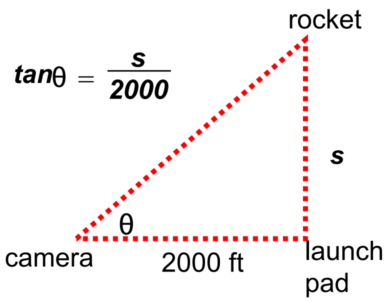
A television camera at ground level is filming the lift-off of a space shuttle that is rising vertically according to the position equation

$s = 50t^2$

where s is measured in feet and t is measured in seconds.

The camera is 2000 feet from the launch pad.

Find the rate of change in the angle of elevation of the camera at 10 seconds after lift-off.



Example 6 - The Velocity of a Piston

In the engine, a 7-inch connecting rod is fastened to a crank of radius *3* inches. The crankshaft rotates counterclockwise at a constant rate of *200* revolutions per minute.

Find the velocity of the piston when $\theta = \pi/3$.