**Why are these functions NOT differentiable at the given point?**

$y_1 = \sqrt[3]{(x+2)^2} - 1 \qquad x = -2$ cusp

$y_2 = \sqrt[3]{x+2} - 1 \qquad x = -2$ Vertical T. L.

$y_3 = \dfrac{1}{\sqrt[3]{x+2}} - 1 \qquad x = -2$ DISC

**Why are these functions NOT differentiable at the given point?**

$$y_4 = \begin{cases} (x+2)^2 - 1, & x \geq -1 \\ 4 + 2x, & x < -1 \end{cases}$$ $(-1,0)$ $(-1,2)$ > DISC @ X = -1

$x^2+4x+4-1$

cont.

$$y_5 = \begin{cases} (x+2)^2 - 1, & x \geq -2 \\ 5 + 3x, & x < 2 \end{cases}$$ $(-2,-1)$ $(-2,-1)$

$$y' = \begin{cases} 2x+4 \\ 3 \end{cases}$$ at $x=-2$: $0$, $3$

POINT

$$y_6 = -(x-2)^{-2}$$ @ X = 2

$-\dfrac{1}{(X-2)^2}$ DISC

**For** $f(x)=x^4-3x^3+2x+1$ **, find the x-value where** ***m = 3***

Slope of T.L. = 3

$f'(x)=4x^3-9x^2+2$

$3=4x^3-9x^2+2$

$0=4x^3-9x^2-1$

Calc. $\rightarrow$ $x \approx 2.297$

**Find the rate of change when $x = 2$ for the given function**

slope

$$f(x) = 3x(3x-4)^2 + (2x-1) + 25$$

$$f(x) = 3x(9x^2 - 24x + 16) + 2x + 24$$

$$f(x) = 27x^3 - 72x^2 + 48x + 2x + 24$$

$$f(x) = 27x^3 - 72x^2 + 50x + 24$$

$$f'(x) = 81x^2 - 144x + 50$$

$$f'(2) = 81(2)^2 - 144(2) + 50$$

$$\boxed{f'(2) = 86}$$

**Find the derivative.**

$$y = \frac{x^2}{\sin x}$$

$$y' = \frac{\sin x(2x) - x^2(\cos x)}{(\sin x)^2}$$

$$y' = \frac{x(2\sin x - x\cos x)}{\sin^2 x}$$

**Find the derivative.**

$$y=\left(x^3+\frac{1}{x^3}\right)\left(x^2-3\right)$$

$$y = x^5 - 3x^3 + x^{-1} - 3x^{-3}$$

$$y' = 5x^4 - 9x^2 - x^{-2} + 9x^{-4}$$

$$y' = 5x^4 - 9x^2 - \frac{1}{x^2} + \frac{9}{x^4}$$

**Find the derivative at the given point using the alternate definition of derivative**

$$y = 3x^2 - x + 1 \quad at \quad x = \underset{c}{2}$$

$$\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

$$\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$$

$$\lim_{x \to 2} \frac{3x^2 - x + 1 - 11}{x - 2}$$

$$\lim_{x \to 2} \frac{3x^2 - x - 10}{x - 2}$$

$$\lim_{x \to 2} \frac{(3x+5)\cancel{(x-2)}}{\cancel{x-2}}$$

$$\lim_{x \to 2} 3x + 5$$

$$3(2) + 5$$

$$6 + 5$$

$$\boxed{11}$$

**Find the derivative**

$$f(x)=\frac{x^2-4}{\sqrt{x}}$$

$$x^{\frac{1}{2}}$$

$$f(x)= x^{\frac{3}{2}}-4x^{-\frac{1}{2}}$$

$$f'(x)=\frac{3}{2}x^{\frac{1}{2}}+2x^{-\frac{3}{2}}$$

$$f'(x)=\frac{3x^{\frac{1}{2}}x^{\frac{3}{2}}}{2\,x^{\frac{3}{2}}}+\frac{2\cdot 2}{x^{\frac{3}{2}}\cdot 2}$$

$$f'(x)=\frac{3x^2+4}{2x\sqrt{x}}$$

If $f(\overset{x}{2}) = \overset{y}{5}$ and $f'(2) = \overset{m}{-\frac{1}{2}}$

**Find the equation of the tangent line at *x* = *2***

$$y - 5 = -\frac{1}{2}(x - 2)$$

**A ball is dropped from 300 feet.**
**Find the velocity when it hits the ground.**

$$s(t) = -16t^2 + 300$$

$$0 = -16t^2 + 300$$

$$\frac{300}{16} = t^2$$

$$\frac{\sqrt{300}}{4} = t \qquad t \approx 4.330$$

$$v(t) = -32t$$

$$v\left(\frac{\sqrt{300}}{4}\right) = -32\left(\frac{\sqrt{300}}{4}\right)$$

$$\boxed{\approx -138.564 \text{ ft/sec}}$$

**Given** $y = \dfrac{x^2 - 1}{x^2 + 1}$ **Find** $y'$

$$y' = \frac{(x^2+1)(2x) - (x^2-1)(2x)}{(x^2+1)^2}$$

$$y' = \frac{\cancel{2x^3} + 2x - \cancel{2x^3} + 2x}{(x^2+1)^2}$$

$$\boxed{y' = \frac{4x}{(x^2+1)^2}}$$

**Given** $f(x) = x^2 + 2\tan x$ $\quad y = \frac{\pi^2}{16} + 2$

**Find the equation of the tangent line at** $x = \frac{\pi}{4}$

$f'(x) = 2x + 2\sec^2 x$

$f'\left(\frac{\pi}{4}\right) = 2\left(\frac{\pi}{4}\right) + 2\sec^2 \frac{\pi}{4}$

$= \frac{\pi}{2} + 2\left(\frac{2}{\sqrt{2}}\right)^2$

$= \frac{\pi}{2} + 2\left(\frac{4}{2}\right)$

$= \frac{\pi}{2} + 4 \rightarrow$ slope of T.L @ $x = \frac{\pi}{4}$

$$\boxed{y - \left(\frac{\pi^2}{16} + 2\right) = \left(\frac{\pi}{2} + 4\right)\left(x - \frac{\pi}{4}\right)}$$

$3x^{\frac{1}{2}}$

**Given** $f(x) = 3\sqrt{x} + 2x^2$

**Find** $f''(4)$

$f'(x) = \frac{3}{2}x^{-\frac{1}{2}} + 4x$

$f''(x) = -\frac{3}{4}x^{-\frac{3}{2}} + 4$

$f''(x) = \frac{-3}{4x^{\frac{3}{2}}} + 4$

$f''(4) = \frac{-3}{4(4)^{3/2}} + 4$

$f''(4) = \frac{-3}{4(8)} + 4$

$f''(4) = -\frac{3}{32} + 4$

$\boxed{f''(4) = \frac{125}{32}}$

**Given** $f(x) = 5\sec x \tan x$

**Find** $f'(x)$

$$f'(x) = 5\left[\sec x(\sec^2 x) + \tan x(\sec x \tan x)\right]$$

$$\boxed{f'(x) = 5\sec x\left[\sec^2 x + \tan^2 x\right]}$$

**Given** $v_0 = -50\, ft/\sec$

$s_0 = 400\, ft$

**Find the velocity after the object has fallen 150 feet.**

$s(t) = -16t^2 - 50t + 400$

after falling 150'

$250 = -16t^2 - 50t + 400$

$0 = -16t^2 - 50t + 150$

$t = 1.875$

$v(t) = -32t - 50$

$\boxed{v(1.875) = -110\ ft/sec}$

**Given the function** $y = x^3 + x$ .
**Find the points on the curve that have a tangent line with a slope of 3**

$y' = 3x^2 + 1$

$3 = 3x^2 + 1$

$2 = 3x^2$

$\frac{2}{3} = x^2$

$\pm\sqrt{\frac{2}{3}} = x$

$\boxed{\left(-\sqrt{\frac{2}{3}}, -1.361\right) \left(\sqrt{\frac{2}{3}}, 1.361\right)}$