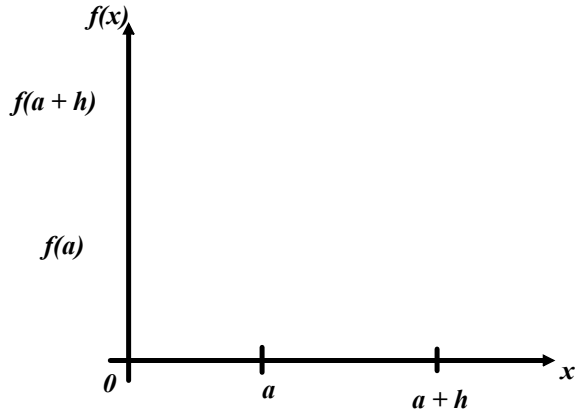
15.2 Difference Ratio

$$\text{Average Speed} \quad - \quad \frac{\Delta \text{value of the function}}{\Delta \text{ value in time}}$$

Average Rate of Change in value of a function f with respect to x

$$\frac{\Delta \text{value of function}}{\Delta \text{ in variable}} \rightarrow \frac{f(a+h) - f(a)}{(a+h) - a} = \frac{f(a+h) - f(a)}{h} \qquad h \neq 0$$



Rate of Change of f at $x = a$

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Difference Quotient

$$\frac{f(a+h) + f(a)}{h}$$

Derivative of f at $x = a$

$$f'(x) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

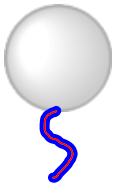
Limit as $h \to 0$ of the difference quotient is called the derivative of f at x = a

A function is differentiable if the derivative of the function exists at each value in its domain.
A function is differentiable on an interval if the derivative of the function exists at each point on the interval

Blow up a balloon.

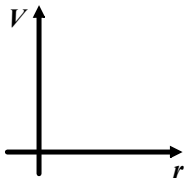
r = radius in cm
V = volume in cm^3

$$V = \frac{4}{3}\pi r^3$$



When r changes → V changes

Find the rate at which V is changing with respect to r when $r = 2$



$$\frac{\Delta \text{volume}}{\Delta \text{ radius}}$$

A farmer estimates that if harvests his crop now he will get 50 bushels per acre which he can sell for $1.00 per bushel. Past experience suggests that his crop will increase at the rate of 5 bushels per week, but the price will probably decline at the rate of 5 cents per bushel per week. However, he can wait no longer than 6 weeks or his crop may be endangered. When should he harvest his crop so that he gets the maximum amount?

A = amount
w = # of weeks he should wait

$$A = (50 + 5w) (1.00 - .05w)$$

$$A = 50 + 5w - 2.5w - .25w^2$$
$$A = .25(200 + 20w - 10w - w^2)$$
$$A = .25(200 + 10w - w^2)$$