Chapter 4 - Trig Functions

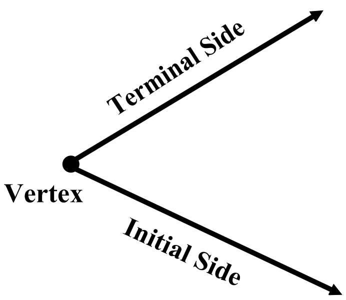
What will you learn?

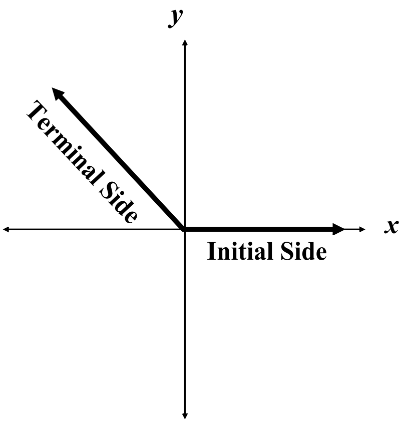
- **To *describe an angle***
- **To *convert* between *degree and radian measure***
- **To identify a *unit circle***
- **To describe a unit circle's *relationship to real numbers***
- **To *evaluate trig functions* on any angle**
- **To use *fundamental trig identities***
- **To sketch *graphs of trig functions***
- **To evaluate *inverse trig functions***
- **To evaluate the *composition of trig functions***
- **To use trig function to model / solve *real-life problems***

4.1 Radian & Degree Measure

Trigonometry - (Greek) "measurement of triangles"

developed over time to study of functions with the set of real numbers as domain

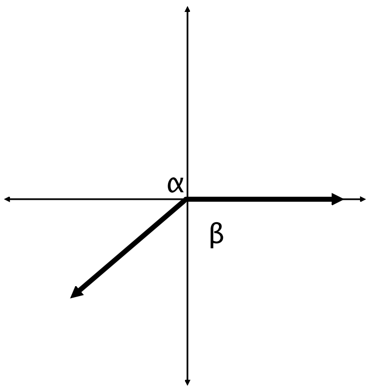




Angle - rotating a ray about its endpoint
Initial Side - starting position
Terminal Side- position after rotation
Vertex - endpoint of ray

Standard Position - initial side of angle coincides with *x*-axis
Positive Angles- counterclockwise rotation
Negative Angles- clockwise rotation

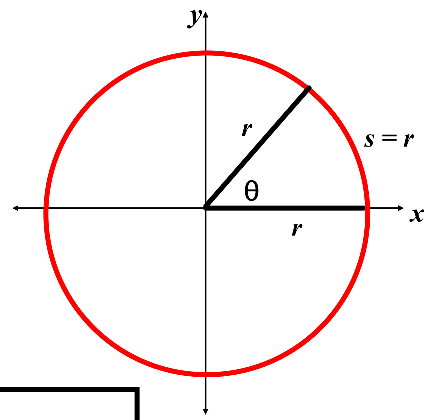
Coterminal Angles



Radian Measure

Useful for calculus!

Central Angle of a Circle



Arc length = radius when θ *= 1 radian*

Definition of Radian

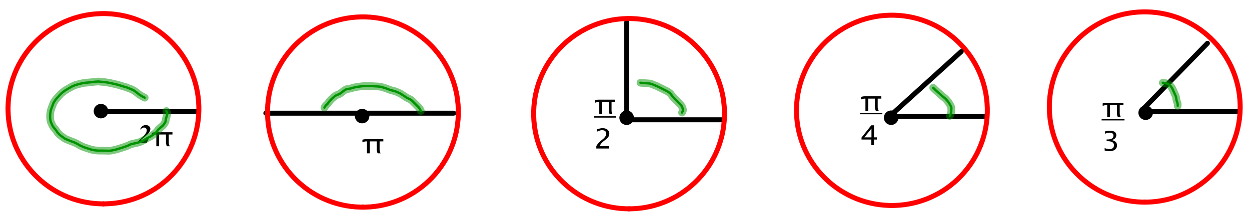
One radian is the measure of a central angle θ that intercepts an arc *s* equal in length to the radius *r* of the circle.

Circumference of a circle = $2\pi r$ *units*

2π *radians* $= 360^0$

π *radians* $= 180^0$

$\pi/2$ *radians* $= 90^0$

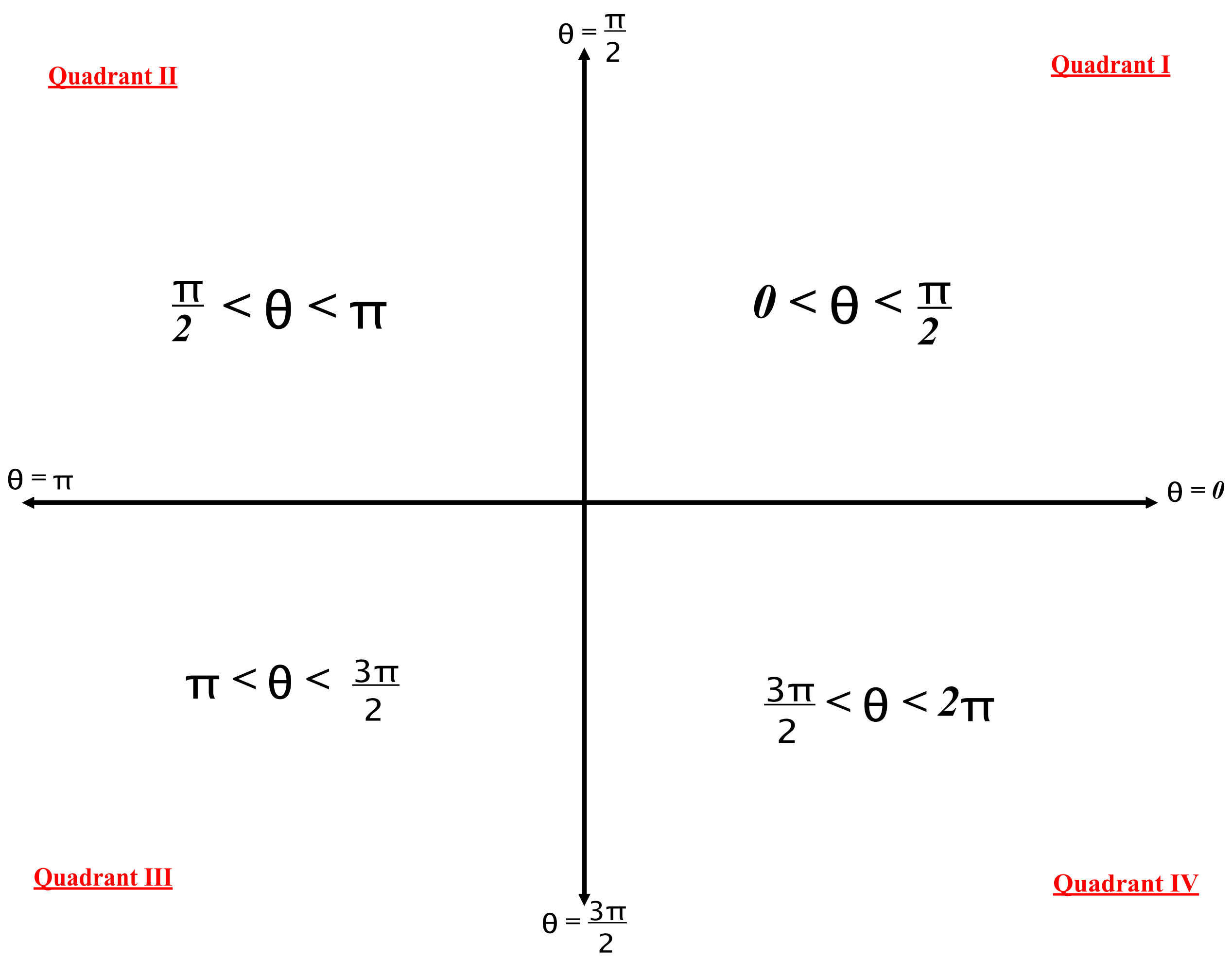


1 revolution $= 2\pi$ *radians*

$\frac{1}{2}$ *revolution* $= \frac{2\pi}{2} = \pi$ *radians*

$\frac{1}{4}$ *revolution* $= \frac{2\pi}{4} = \frac{\pi}{2}$ *radians*

$\frac{1}{6}$ *revolution* $= \frac{2\pi}{6} = \frac{\pi}{3}$ *radians*


Quadrant II
Quadrant I
θ = π/2
π/2 < θ < π
0 < θ < π/2
θ = π
θ = 0
π < θ < 3π/2
3π/2 < θ < 2π
Quadrant III
Quadrant IV
θ = 3π/2

Coterminal Angles - have same initial and terminal sides

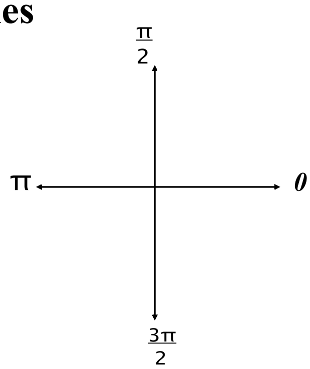
0 & 2π $\qquad\qquad$ $\frac{\pi}{6}$ & $\frac{13\pi}{6}$

You can find coterminal angles by adding or subtracting 2π to a given angle θ

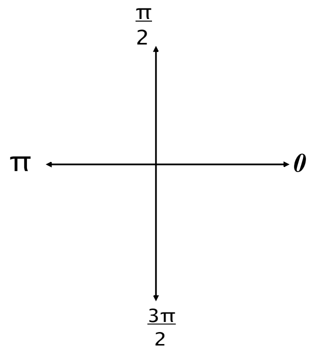
Eg) $\theta = \frac{\pi}{6}$ $\qquad$ Coterminal angles : $\frac{\pi}{6} + 2n\pi$ $\quad$ *$n \in$ integers*

Example 1 - Sketching and Finding Coterminal Angles

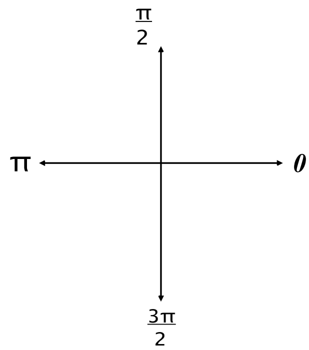
a.) for the positive angle $\theta =$ $\qquad$, subtract 2π



b.) for the positive angle $\theta =$ $\qquad$, subtract 2π



c.) for the negative angle $\theta =$ $\qquad$, add 2π



See p. 255; problem 13

Given 2 positive angles α and βs

Complementary Angles - sum = $\frac{\pi}{2}$

Supplementary Angles - sum = π

Example 2 - Complementary & Supplementary Angles

If possible, find the complement and supplement of

a.) $\frac{2\pi}{5}$

b.) $\frac{4\pi}{5}$

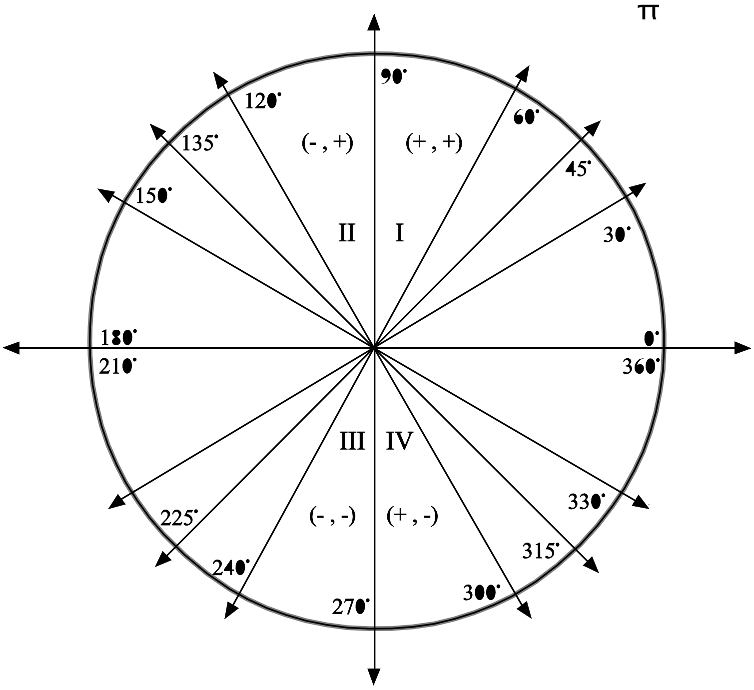
See p. 256; problem 17

Degree Measure

$$1^0 = \frac{1}{360} \text{ revolution}$$

$$360^0 = 2\pi \text{ rad}$$
$$180^0 = \pi \text{ rad}$$

$$1^0 = \frac{\pi}{180} \text{ rad}$$
$$1 \text{ rad} = \frac{180^0}{\pi}$$



Conversion between Degrees and Radians

Degrees to Radians	Radians = degrees • $\frac{\pi \text{ rad}}{180^0}$
Radians to Degrees	Degrees = radians • $\frac{180^0}{\pi \text{ rad}}$

Example 3 - Converting from Degrees to Radians

a.) 135^0

b.) 540^0

c.) -270^0

Example 4 - Converting from Radians to Degrees

a.) $\frac{-\pi}{2}$ rad

b.) π rad

c.) $\frac{9\pi}{2}$ rad

See p. 256; exercises 45 & 51

Linear and Angular Speed

The *radian measure formula* $\theta = s / r$ can be used to measure arc length along a circle.

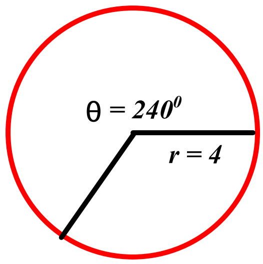
Specifically, for a circle of radius *r*,
and a central angle θ (radians) intercepts an arc
of length *s* given by :

$$s = r\,\theta$$

Length of circular arc

Example 5 - Finding Arc Length

A circle has a radius of 4 inches.
Find the length of the arc intercepted by a central angle of 240^0.



See p. 257; exercise 85

Linear and Angular Speed

Consider a particle moving at a constant speed along a circular arc of radius *r*.
If *s* is the length of the arc travelled in time *t*,
then the linear speed of the particle is

$$\text{Linear Speed} = \frac{\text{arc length}}{\text{time}} = \frac{s}{t}$$

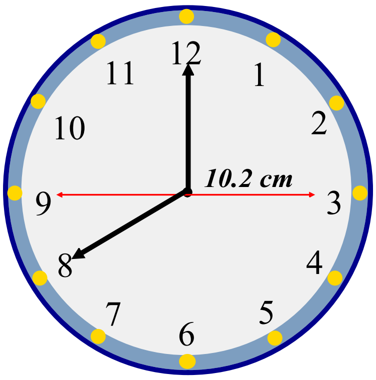
Moreover, if θ is the angle (radians) corresponding to the arc length *s*,
then the angular speed of the particle is

$$\text{Angular Speed} = \frac{\text{central angle}}{\text{time}} = \frac{\theta}{t}$$

Linear speed measures how fast the particle moves.
Angular speed measures how fast the particle changes.

Example 6 - Finding Linear Speed

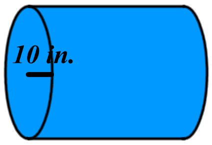
The second hand of a clock is 10.2 centimeters long. Find the linear speed of the tip of the second hand.



See p. 258; exercise 96

Example 7 - Finding the Angular and Linear Speed

A lawn roller with a 10 inch radius makes 1.2 revolutions per second.



a.)   Find the angular speed of the roller in radians per second.

b.) Find the speed of the tractor that is pulling the roller.