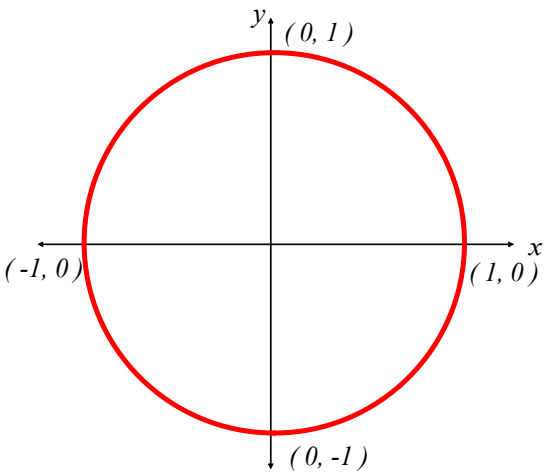
4.2 Trigonometric Functions : The Unit Circle

$$x^2 + y^2 = 1$$



Imagine that the real number system is wrapped around the circle, with positive numbers corresponding to a counterclokwise wrapping and negative numbers corresponding to ta clockwise wrapping.

The Trig Functions

sine	cosecant	sin	csc
cosine	secant	cos	cos
tangent	cotangent	tan	cot

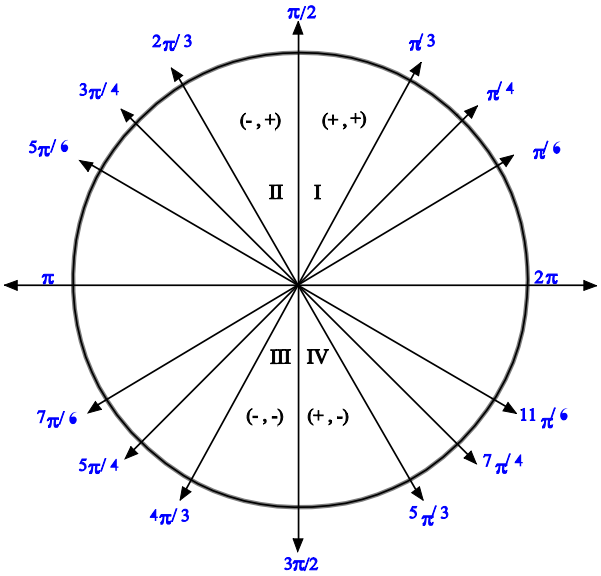
Definition of Trig Functions

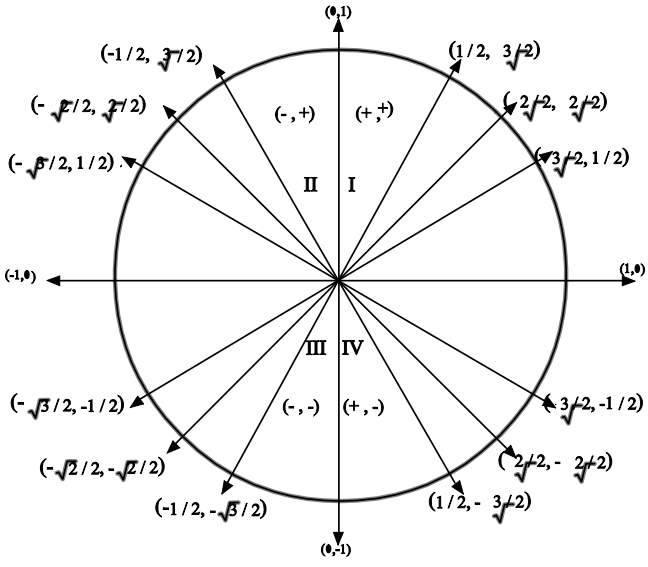
Let t be a real number and let (x, y) be the point on the unit circle corresponding to t

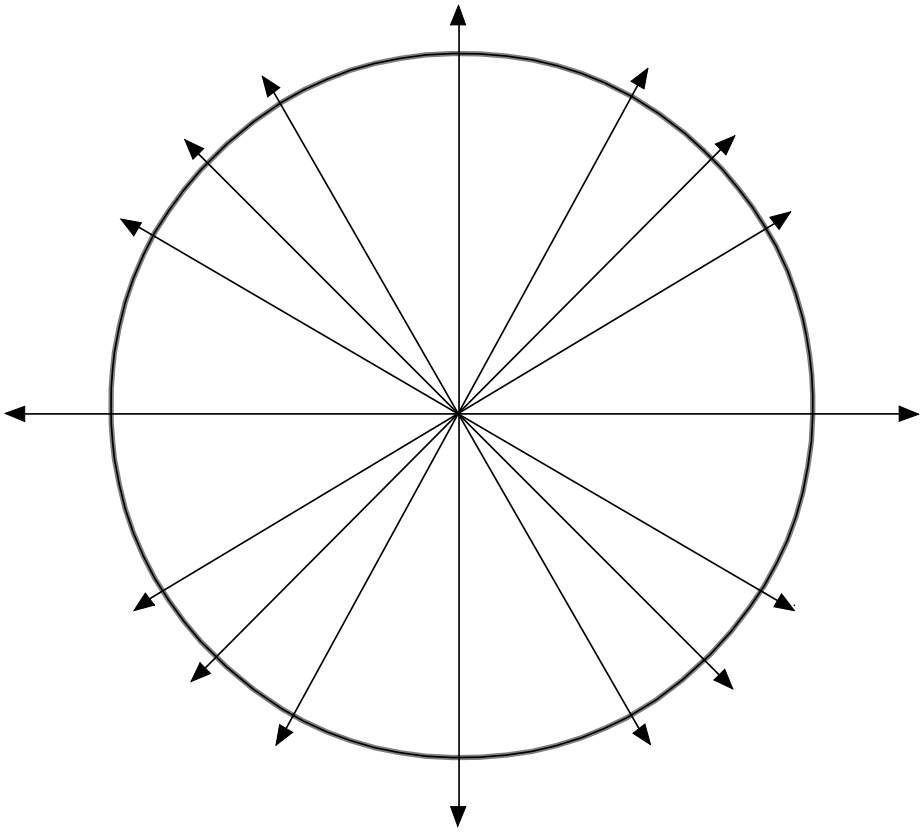
$$\sin t = y \qquad \csc t = \frac{1}{y} \quad y \neq 0$$

$$\cos t = x \qquad \sec t = \frac{1}{x} \quad x \neq 0$$

$$\tan t = \frac{y}{x} \quad x \neq 0 \qquad \cot t = \frac{x}{y} \quad y \neq 0$$







Example 1 - Evaluating Trig Functions

Evaluate the six trig functions at each real number

a.) $t = \frac{\pi}{6} \rightarrow (x, y) = (\sqrt{3}/2,\ 1/2)$

$\sin \frac{\pi}{6} =$ $\qquad$ $\csc \frac{\pi}{6} =$

$\cos \frac{\pi}{6} =$ $\qquad$ $\sec \frac{\pi}{6} =$

$\tan \frac{\pi}{6} =$ $\qquad$ $\cot \frac{\pi}{6} =$

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b.) $t = \frac{5\pi}{4} \rightarrow (x, y) = (-\sqrt{2}/2,\ -\sqrt{2}/2)$

$\sin \frac{5\pi}{4} =$ $\qquad$ $\csc \frac{5\pi}{4} =$

$\cos \frac{5\pi}{4} =$ $\qquad$ $\sec \frac{5\pi}{4} =$

$\tan \frac{5\pi}{4} =$ $\qquad$ $\cot \frac{5\pi}{4} =$

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c.) $t = 0 \rightarrow (x, y) = (1, 0)$

$\sin 0 =$ $\qquad$ $\csc 0 =$

$\cos 0 =$ $\qquad$ $\sec 0 =$

$\tan 0 =$ $\qquad$ $\cot 0 =$

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d.) $t = \pi \rightarrow (x, y) = (-1, 0)$

$\sin \pi =$ $\qquad$ $\csc \pi =$

$\cos \pi =$ $\qquad$ $\sec \pi =$

$\tan \pi =$ $\qquad$ $\cot \pi =$

See p. 264; exercise 23

Example 2 - Evaluating Trig Functions

Evaluate the six trig functions at $t = -\frac{\pi}{3}$

Moving clockwise around the unit circle $(x, y) = (1/2, -\sqrt{3}/2)$

$\sin -\frac{\pi}{3} =$ $\quad$ $\csc -\frac{\pi}{3} =$

$\cos -\frac{\pi}{3} =$ $\quad$ $\sec -\frac{\pi}{3} =$

$\tan -\frac{\pi}{3} =$ $\quad$ $\cot -\frac{\pi}{3} =$

See p. 264; exercise 27

Domain and Period of the Sine & Cosine

Domain of Sine & Cosine → **Real Numbers**

Range of Sine & Cosine → $[-1, 1]$

Periodic Function

Repetitive or cyclical

Adding 2π to each value of t in the interval $[0, 2\pi]$ completes a second revolution around the unit circle.

The values of $\sin(t + 2\pi)$ correspond to the values of $\sin t$

The values of $\cos(t + 2\pi)$ correspond to the values of $\cos t$

$$\sin(t + 2\pi n) = \sin t$$

$$\cos(t + 2\pi n) = \cos t$$

$$f(t + c) = f(t)$$

Example 3 - Using the Period to Evaluate the Sine Function

Evaluate $\sin \frac{13\pi}{6}$

See p. 264; exercise 31

Remember : A function f is even if $f(-t) = f(t)$.
A function f is odd if $f(-t) = -f(t)$.

Even & Odd Trig Functions

The cosine and secant functions are EVEN

$\cos(-t) = \cos t$ $\qquad$ $\sec(-t) = \sec t$

The sine, cosecant, tangent & cotangent functions are ODD

$\sin(-t) = -\sin t$ $\qquad$ $\csc(-t) = -\csc t$

$\tan(-t) = -\tan t$ $\qquad$ $\cot(-t) = -\cot t$

Using a Calculator to evaluate trig functions

Set Mode to radian or degrees

use $\boxed{x^{-1}}$ **button**

use parentheses around all fractions