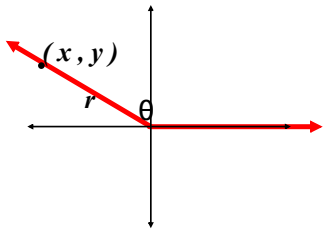
4.4 Trig Functions of Any Angle

Definitions of Trig Functions of Any Angle

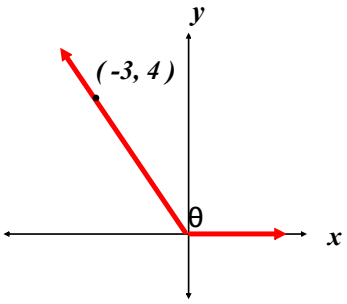
Let θ be an angle in standard position with *(x, y)* a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$.



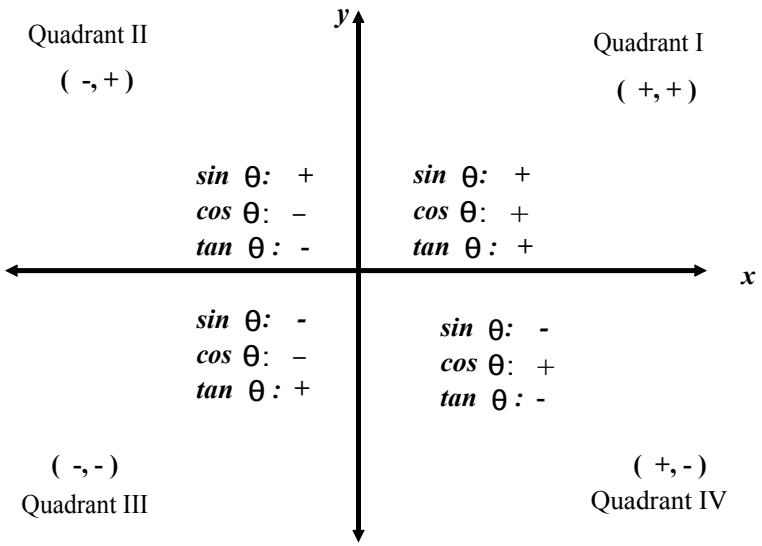
$\sin\theta = \frac{y}{r}$		$\cos\theta = \frac{x}{r}$	
$\tan\theta = \frac{y}{x}$	$x \neq 0$	$\cot\theta = \frac{x}{y}$	$y \neq 0$
$\sec\theta = \frac{r}{x}$	$x \neq 0$	$\csc\theta = \frac{r}{y}$	$y \neq 0$

Example 1 - Evaluating Trig Functions

Let *(-3, 4)* be a point on the terminal side of θ.
Find the sine, cosine & tangent of θ.



See p. 284; exercise 1



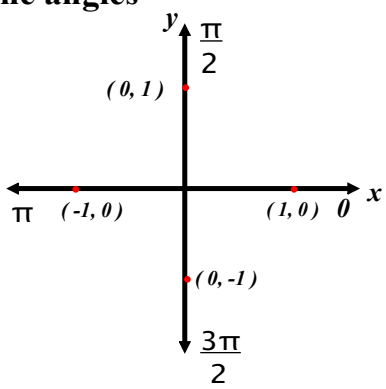
Example 2 - Evaluating Trig Functions

Given $\tan\theta = -5/4$ **and cos > 0, find** $\sin\theta$ **and** $\sec\theta$.

See p. 284; exercise 19

Example 3 - Trig Functions of Quadrant Angles

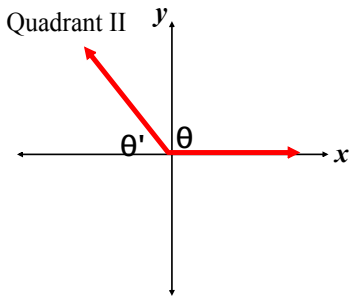
Evaluate the sine and cosine functions at the angles
0, π/2, π, 3π/2



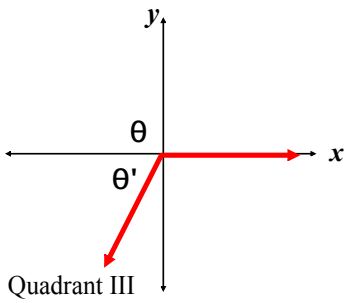
See p. 285; exercise 29

Reference Angles

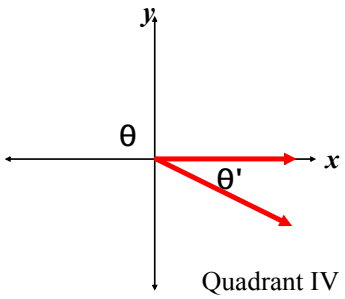
Let θ be an angle in standard position.
Its reference angle is the acute angle θ' formed by the terminal side of θ and the horizontal axis.



$\theta' = \pi - \theta$ **(radians)**
$\theta' = 180^0 - \theta$ **(degrees)**



$\theta' = \theta - \pi$ **(radians)**
$\theta' = \theta - 180^0$ **(degrees)**



$\theta' = 2\pi - \theta$ **(radians)**
$\theta' = 360^0 - \theta$ **(degrees)**

Example 4 - Finding Reference Angles

Find the reference angle θ'

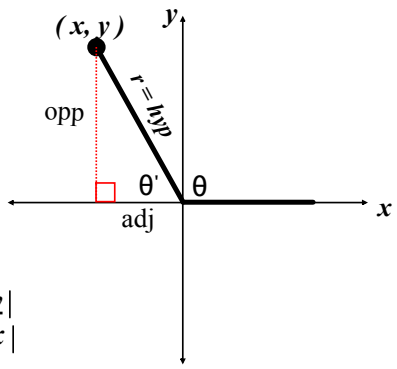
a.) $\theta = 300^0$

b.) $\theta = 2.3$

c.) $\theta = -135^0$

Trig Functions of Real Numbers

$\sin\theta = \frac{y}{r}$ $\quad\quad$ $\tan\theta = \frac{y}{x}$



For the triangle with acute angle θ' and sides of lengths $|x|$ and $|y|$, you have

$\sin\theta' = \frac{opp}{hyp} = \frac{|y|}{r}$ $\quad\quad$ $\tan\theta' = \frac{opp}{adj} = \frac{|y|}{|x|}$

Therefore, the sin θ and $\sin\theta'$ are equal, *except possible in sign*.
The same is true for tan and all other four trig functions.
In all cases, the sign of the function can be determined by the quadrant in which θ lies.

To find the value of a trig function of any angle θ:

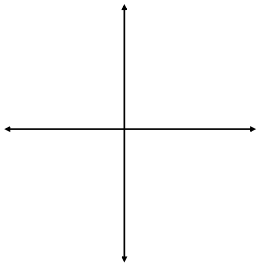
1. Determine the function value for the associated reference angle θ'
2. Depending on the quadrant in which θ lies, affix the appropriate sign to the function value.

θ	0	30	45	60	90
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$

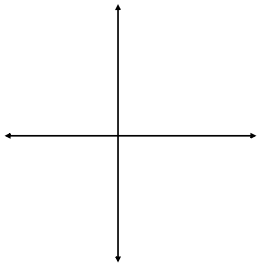
Reverse the order to get the cosine!

Example 5 - Trig Functions of Nonacute Angles

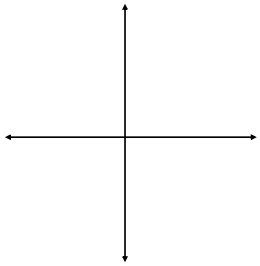
a.) $\cos \frac{4\pi}{3}$



b.) $\tan(-210^0)$



c.) $\csc \frac{11\pi}{4}$



Example 6 - Using Trig Identities

Let θ be an angle in QII *s.t.* $\sin\theta = 1/3$. Find $\cos\theta$ by using trig identities.

Example 7 - Using a Calculator

Use a calc. to evaluate each trig. function.

a.) $\cot 410^0$

b.) $\sin(-7)$

c.) $\sec \pi/9$

Set you calc to degree mode.
Enter tan 90.
What happens? Why?
Now wet calc to radian mode.
Enter tan $\pi/2$.
Explain the graphing utility's answer.