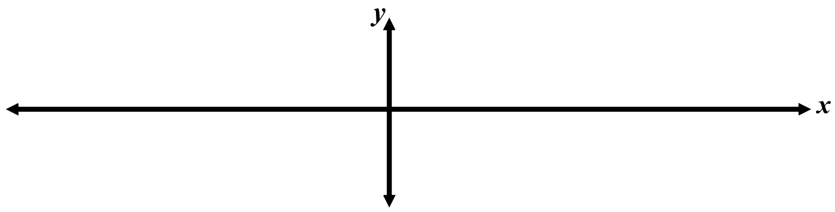
4.7 Inverse Trig Functions

Remember - For a function to have an inverse....it must be 1:1
(Horizontal Line Test)



Obviously, $y = \sin x$ **does NOT pass the test**

However, if you *restrict the domain* to the interval $-\pi/2 \leq x \leq \pi/2$**, the following properties hold:**

1. $y = \sin x$ **is increasing**
2. **the *range* of** $y = \sin x$ **is [-1,1]**
3. $y = \sin x$ **is 1:1**

Inverse Sine Function → $y = \arcsin x$ or $y = \sin^{-1} x$

"the angle whose sine is *x*"

Note - y = arcsin x → sin y = x

Definition of Inverse Sine Function

y = arcsin x iff sin y = x

where $-1 \leq x \leq 1$ **and** $-\pi/2 \leq y \leq \pi/2$
Domain : [-1,1] **Range:** $[-\pi/2, \pi/2]$

Example 1 - Evaluating the Inverse Sine Function

If possible, find the exact value of

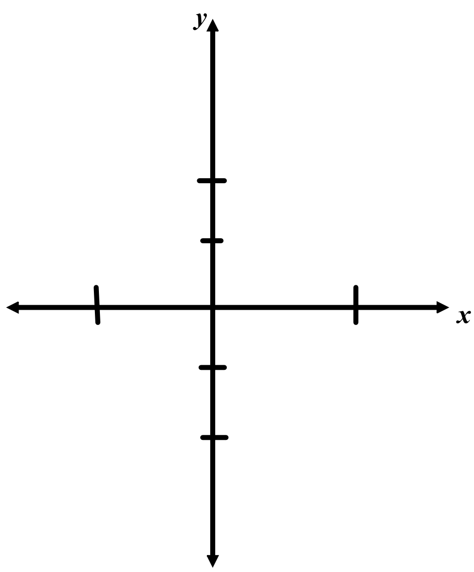
a.) arcsin (-1/2)

b.) $\sin^{-1}(\sqrt{3}/2)$

c.) $\sin^{-1} 2$

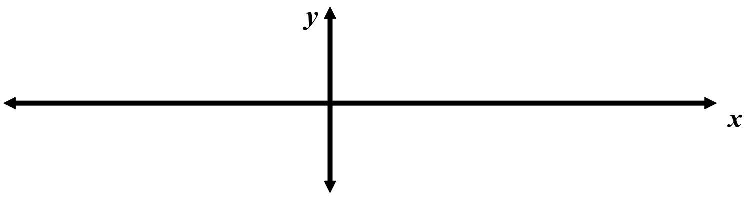
Example 2 - Graphing the Arcsine Function

Sketch the graph of $y = \arcsin x$ by hand

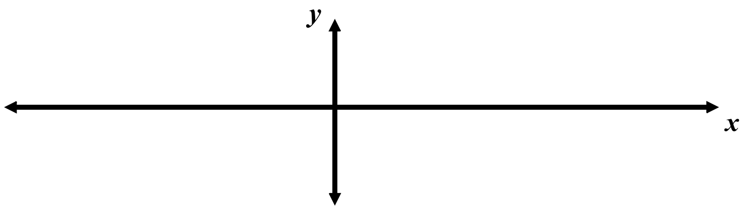


Other Inverse Trig Functions

Cosine is decreasing and 1:1 on the interval $0 \leq x \leq \pi$



Tangent is increasing and 1 : 1 on the interval $-\pi/2 \leq x \leq \pi/2$



Function	Domain	Range
$y = \arcsin x$ iff $\sin y = x$	$-1 \leq x \leq 1$	$-\pi/2 \leq y \leq \pi/2$
$y = \arccos x$ iff $\cos y = x$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$
$y = \arctan x$ iff $\tan y = x$	$-\infty < x < \infty$	$-\pi/2 \leq y \leq \pi/2$

Example 3 - Evaluating Inverse Trig Functions

Find the exact value.

a.) $arccos\ \sqrt{2}/2$

b.) $cos^{-1}(-1)$

c) $arctan\ 0$

d.) $tan^{-1}(-1)$

Example 4 - Calculators & Inverse Trig Functions

Use a calc. to approximate the value, if possible

Mode

a.) ***arctan (-8.45)***

b.) $\boldsymbol{sin^{-1}\ (0.2447)}$

c.) ***arccos 2***

Tech Tip - if you had set the calc to degree mode,
the display sould have been in degrees rather than radians.

By definition, the values of inverse trig functin are always in radians

Compositions of Functions

Remember....... $f(f^{-1}(x)) = x$ **and** $f^{-1} f(x) = x$

Inverse Properties

If $-1 \leq x \leq 1$ **and** $-\pi/2 \leq y \leq \pi/2$

$\sin(\arcsin x) = x$ **and** $\arcsin(\sin y) = y$

If $-1 \leq x \leq 1$ **and** $0 \leq y \leq \pi$

$\cos(\arccos x) = x$ **and** $\arccos(\cos y) = y$

If x **is a real number and** $-\pi/2 < y < \pi/2$

$\tan(\arctan x) = x$ **and** $\arctan(\tan y) = y$

Example 5 - Using Inverse Properties

If possible, find the exact value:

a.) *tan [arctan (-5)]*

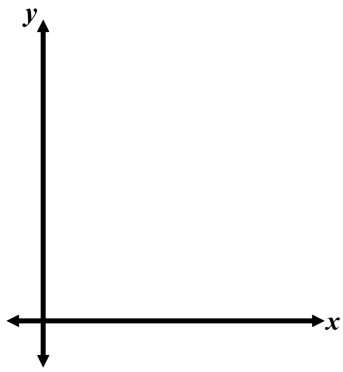
b.) *arcsin (sin 5π/3)*

c.) $\cos(\cos^{-1} \pi)$

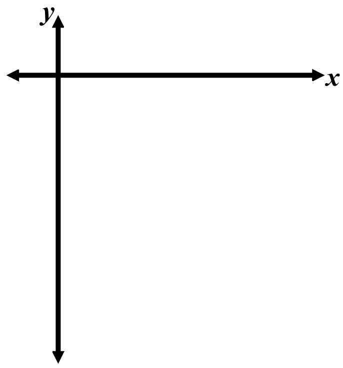
Example 6 - Evaluating Compositions of Functions

Find the exact value:

a.) *tan (arccos 2/3)*



b.) *cos [arcsin (-3/5)]*



Example 7 - Some Problems from Calculus

Write each of the following as an algebraic expression in *x*

a.) $\sin(\arccos 3x), \quad 0 \le x \le 1/3$

b.) $\cot(\arccos 3x), \quad 0 \le x \le 1/3$