Honors – Precal
Worksheet 5.3 – 5.5

Name Key

nd all solutions in $[0,2\pi)$. Show work.

1. $\sin 4x - \sin 2x = 0$

$2(2x)$

$2\sin 2x\cos 2x - \sin 2x = 0$

$\sin 2x\,(2\cos 2x - 1) = 0$

$\sin 2x = 0$ $2\cos 2x - 1 = 0$

$\sin\theta = 0$ $\cos 2x = \frac{1}{2}$

$\theta = 0, \pi$ $\cos\theta = \frac{1}{2}$

$2x = 0, \pi$ $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

$x = 0, \frac{\pi}{2}$ Period $= \pi$ Period $= \pi$ $2x = \frac{\pi}{3}, \frac{5\pi}{3}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

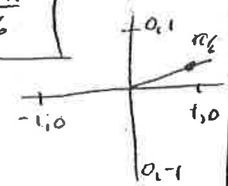
2. $2\cos x - 2\sin 2x = 0$

$2\cos x - 2(2\sin x\cos x) = 0$

$2\cos x\,(1 - 2\sin x) = 0$

$\cos x = 0$ $\sin x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad x = \frac{\pi}{6}, \frac{5\pi}{6}}$



$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $x = \frac{\pi}{6}, \frac{5\pi}{6}$

3. $2\cos^2 x - \cos x = 1$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$\cos x = \frac{-1}{2}$ $\cos x = 1$

$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3} \qquad x = 0}$

$x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$

4. $\cos(x + \frac{\pi}{4}) + \cos(x - \frac{\pi}{4}) = 1$

$2\cos\left(\frac{x+\frac{\pi}{4} + x - \frac{\pi}{4}}{2}\right)\cos\left(\frac{x+\frac{\pi}{4} - (x - \frac{\pi}{4})}{2}\right) = 1$

$2\cos\left(\frac{2x}{2}\right)\cos\left(\frac{\frac{2\pi}{4}}{2}\right) = 1$

$2\cos x \cos\frac{\pi}{4} = 1$

$\cancel{2}\cos x\left(\frac{\sqrt{2}}{\cancel{2}}\right) = 1$

$\cos x\,(\sqrt{2}) = 1$

$\cos x = \frac{1}{\sqrt{2}}$

$x = \frac{\pi}{4}, \frac{7\pi}{4}$ $\cos x = \frac{\sqrt{2}}{2}$

$x = \pi/4, 7\pi/4$

5. $\cos^2 2x - \sin^2 2x = -1$

$\cos 4x = -1$

Let $\theta = 4x$ Period $\frac{2\pi}{4} = \frac{\pi}{2}$

$\cos\theta = -1$

$\theta = \pi$ $x = \frac{\pi}{4} + \frac{\pi}{2}n$

$4x = \pi$ $\frac{\pi}{4} + \frac{2\pi}{4} = \frac{3\pi}{4}$

$x = \frac{\pi}{4}$ $\frac{5\pi}{4}$

$\frac{7\pi}{4}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

6. $2\cos^2 3x - 1 = 0$ Let $\theta = 3x$

$\cos^2 3x = \frac{1}{2}$

$\cos^2\theta = \frac{1}{2}$ Period $= \frac{2\pi}{3}$ $\frac{8\pi}{12}$

$\cos\theta = \frac{\pm 1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$3\theta = \frac{\pi}{4}$ $3\theta = \frac{3\pi}{4}$ $3\theta = \frac{5\pi}{4}$ $3\theta = \frac{7\pi}{4}$

$\theta = \frac{\pi}{12} + \frac{8\pi}{12}$ $\theta = \frac{\pi}{4}$ $\frac{5\pi}{12}$ $\frac{7\pi}{12}$

$x = \frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{15\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{21\pi}{12}$

$2\pi = \frac{24\pi}{12}$

Prove the identity.

7. $\sin(x-\frac{3\pi}{2})=\cos x$	8. $2\sin y\sec(2y)\cos y=\tan(2y)$
$(0,-1)$	
$\sin x\cos\frac{3\pi}{2}-\cos x\sin\frac{3\pi}{2}=$	$(2\sin y\cos y)\frac{1}{\cos 2y}=$
$\sin x(0)-\cos x(-1)=$	$\sin 2y\cdot\frac{1}{\cos 2y}=$
$\cos x=\cos x$	$\frac{\sin 2y}{\cos 2y}=$
	$\tan 2y=\tan 2y$
9. $\frac{\cos 3x-\cos x}{\sin 3x-\sin x}=-\sin 2x\sec 2x$	10. $\tan\frac{u}{2}=\csc u-\cot u$
$\frac{-\cancel{2}\sin 2x\cancel{\sin x}}{\cancel{2}\cos 2x\cancel{\sin x}}$	$\frac{1-\cos u}{\sin u}=$
$-\frac{\sin 2x}{\cos 2x}$	$\frac{1}{\sin u}-\frac{\cos u}{\sin u}=$
$-\sin 2x\sec 2x=-\sin 2x\sec 2x$	$\csc u-\cot u=\csc u-\cot u$
11. $\csc 2x=\frac{\csc x}{2\cos x}$	12. $\cos^2 5x-\cos^2 x=-\sin 4x\sin 6x$
$\frac{1}{\sin 2x}=$	$\frac{1+\cos 10x}{2}-\frac{1+\cos 2x}{2}=$
$\frac{1}{2\sin x\cos x}=$	$\frac{\cos 10x-\cos 2x}{2}=$
$\frac{\csc x}{2\cos x}=\frac{\csc x}{2\cos x}$	$\frac{-\cancel{2}\sin 6x\sin 4x}{\cancel{2}}=$
	$-\sin 4x\sin 6x=-\sin 4x\sin 6x$

Find all solutions.

13. $\sin^2 x \cos x - \cos x = 0$

$\cos x(\sin^2 x - 1) = 0$

$\cos x = 0 \qquad \sin^2 x = 1$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}} \qquad \sin x = \pm 1$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

14. $4\cos^2 x - 2 = 1$

$4\cos^2 x = 3$

$\cos^2 x = \frac{3}{4}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

15. $3\tan^2 x - \sec^2 x - 5 = 0$

$3\tan^2 x - (\tan^2 x + 1) - 5 = 0$

$2\tan^2 x - 6 = 0$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

$\boxed{x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

16. $4\sin^2 2x = 1$

Let $\theta = 2x$

$4\sin^2\theta = 1$

$\sin^2\theta = \frac{1}{4}$

$\sin\theta = \pm\frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$2x = \frac{\pi}{6}$

Period $= \pi$ $\quad x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$

$+\pi \quad +\pi \quad +\pi \quad +\pi$

$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

17. $\cos x + \sin x \tan x = 2$

$\cos x + \sin x \frac{\sin x}{\cos x} = 2$

$\cos x + \frac{\sin^2 x}{\cos x} = 2$

$\cos x + \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x} = 2$

$\cos x + \frac{1}{\cos x} - \cos x = 2$

$\frac{1}{\cos x} = 2$

$\cos x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

18. $\sec 3x = \sqrt{2}$

$\cos 3x = \frac{1}{\sqrt{2}}$

$\cos \theta = \frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}, \frac{7\pi}{4}$

$3x = \frac{\pi}{4}$ $\qquad$ $3x = \frac{7\pi}{4}$

$x = \frac{\pi}{12}$ $\qquad$ $x = \frac{7\pi}{12}$

Period $= \frac{2\pi}{3} = \frac{8\pi}{12}$

$\frac{\pi}{12}$ $\frac{9\pi}{12}$ $\frac{17\pi}{12}$

$x = \frac{\pi}{12}, \frac{3\pi}{4}, \frac{17\pi}{12}$

$x = \frac{7\pi}{12}, \frac{5\pi}{4}, \frac{23\pi}{12}$ $\qquad$ $\frac{7\pi}{12}$ $\frac{15\pi}{12}$ $\frac{23\pi}{12}$ ✓

19. $2\cos^2 x - \sin x - 1 = 0$

$2(1-\sin^2 x) - \sin x - 1 = 0$

$2 - 2\sin^2 x - \sin x - 1 = 0$

$-2\sin^2 x - \sin x + 1 = 0$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}$ $\qquad$ $\sin x = -1$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6} \qquad x = \frac{3\pi}{2}}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

Typo

20. Find the general solution.

$\sin^3 x = \sin x$

$\sin^3 x - \sin x = 0$

$\sin x(\sin^2 x - 1) = 0$

$\sin x = 0$ $\qquad$ $\sin^2 x = 1$

$x = 0, \pi$ $\qquad$ $\sin x = \pm 1$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$0 + 2\pi n$ $\qquad$ $\frac{\pi}{2} + 2\pi n$

$\pi + 2\pi n$ $\qquad$ $\frac{3\pi}{2} + 2\pi n$

$0 + 2\pi n$ $\qquad$ $\frac{\pi}{2} + 2\pi n$

$\pi + 2\pi n$ $\qquad$ $\frac{3\pi}{2} + 2\pi n$