

1) $\csc^2 x = 2$

$$\sec^2 x = \frac{1}{2}$$

$$\sec x = \pm \frac{1}{\sqrt{2}} \text{ or } \pm \frac{\sqrt{2}}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

2) $\sin^2 2x - 1 = 0$

$$\sec^2 2x = 1$$

$$\sec 2x = \pm 1$$

$$2x = \frac{\pi}{2} + 2\pi$$

$$2x = \frac{3\pi}{2} + 2\pi$$

$$x = \frac{\pi}{4} + \pi$$

$$x = \frac{3\pi}{4} + \pi$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

3) $2\cos^2 x - \sin x - 1 = 0$

$$2(1 - \sin^2 x) - \sin x - 1 = 0$$

$$2 - 2\sin^2 x - \sin x - 1 = 0$$

$$-2\sin^2 x - \sin x + 1 = 0$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$$\sin x = \frac{1}{2} \quad \sin x = -1$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6} \quad x = \frac{3\pi}{2}$$

4) $\cos^2 x = 2 - \cos x$

$$\cos^2 x + \cos x - 2 = 0$$

$$(\cos x + 2)(\cos x - 1) = 0$$

$$\cos x = -2$$

$$\cos x = 1$$

$$\downarrow$$

$$\emptyset$$

$$x = 0$$

5) $3\tan^2 x - \sec^2 x - 5 = 0$

$$3\tan^2 x - (1 + \tan^2 x) - 5 = 0$$

$$3\tan^2 x - 1 - \tan^2 x - 5 = 0$$

$$2\tan^2 x - 6 = 0$$

$$2\tan^2 x = 6$$

$$\tan^2 x = 3$$

$$\tan x = \pm\sqrt{3}$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

6) $2\cos^2 3x - 1 = 0$

$$2\cos^2(3x) = 1$$

$$\cos^2(3x) = \frac{1}{2}$$

$$\cos(3x) = \pm \frac{1}{\sqrt{2}} \text{ or } \pm \frac{\sqrt{2}}{2}$$

$$3x = \frac{\pi}{4} + 2\pi$$

$$3x = \frac{3\pi}{4} + 2\pi$$

$$3x = \frac{5\pi}{4} + 2\pi$$

$$3x = \frac{7\pi}{4} + 2\pi$$

$$x = \frac{\pi}{12} + \frac{2\pi}{3}$$

$$x = \frac{\pi}{4} + \frac{2\pi}{3}$$

$$x = \frac{5\pi}{12} + \frac{2\pi}{3}$$

$$x = \frac{7\pi}{12} + \frac{2\pi}{3}$$

$$\frac{9\pi}{12}$$

$$\frac{11\pi}{12}$$

$$\frac{13\pi}{12}$$

$$\frac{15\pi}{12}$$

$$\frac{17\pi}{12}$$

$$\frac{19\pi}{12}$$

$$\frac{21\pi}{12}$$

$$\frac{23\pi}{12}$$