5.4 - Sum & Difference Formulas

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u-v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u-v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

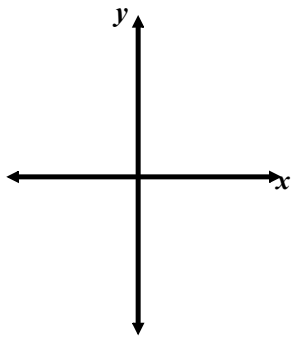
$$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

Example 1 - Evaluating a Trig Function

Find the exact value of $\cos 75^0$

Example 2 - Evaluating a Trig Function

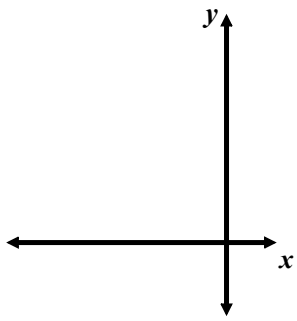
Find the exact value of $\sin \pi/12$



Example 3 - Evaluating a Trig Expression

Find the exact value of $\sin(u + v)$

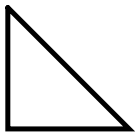
$\sin u = 4/5$, **where** $0 < u < \pi/2$ **and** $\cos v = -12/13$, **where** $\pi/2 < v < \pi$

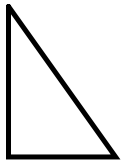


Example 4 - An Application of a Sum Formula

Write as an algebraic expression :

$$\cos (\arctan 1 + \arccos x)$$





Example 5 - Proving a Cofunction Identity

Prove

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

Reduction Formula

Sum & Difference formulas can be used to derive reduction formulas invloving expressions such as :

$\sin(x + n\pi/2)$ $\qquad\qquad$ $\cos(x + n\pi/2)$

$n \in$ *integers*

Example 6 - Deriving Reduction Formulas

Simplify each expression

a.) $\cos(x - 3\pi/2)$

b.) $\tan(x + 3\pi)$

Example 7 - Solving a Trig Equation

Find all solutions in the interval $[0, 2\pi]$

$$\sin\left(x + \pi/4\right) + \sin\left(x - \pi/4\right) = -1$$

Example 8 - An application from Calculus

Verify :

$$\frac{\sin(x+h)-\sin x}{h} = (\cos x)\left(\frac{\sin h}{h}\right) - (\sin x)\left(\frac{1-\cos h}{h}\right) \qquad h \neq 0$$