5.5 - Multiple-Angle & Product-to-Sum Formulas

Double-Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u$$
$$= 2\cos^2 u - 1$$
$$= 1 - 2\sin^2 u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

Example 1 - Solving a Multiple-Angle Equation

Solve

$$2 \cos x + \sin 2x = 0$$

Example 2 - Using Double-Angle Formulas to Analyze Graphs

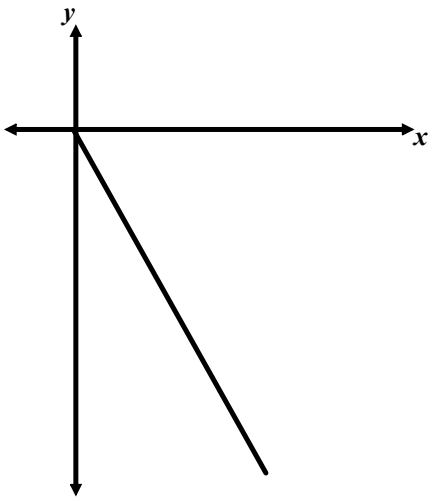
Analyze the graph in the interval $[0, 2\pi]$

$$y = 4\cos^2 x - 2$$

Example 3 - Evaluating Functions Involving Double Angles

Use the following to find ***sin 2x, cos 2x,*** **and** ***tan 2x***

$$\cos x = \frac{5}{13} \qquad 3\pi/2 < x < 2\pi$$



Double Angle Formulas - **not** **restricted to** $2x$ **and** x
$4x$ **and** $2x$ **or** $6x$ **and** $3x$ **are valid also**

ie.) $\sin 4x = 2 \sin 2x \cos 2x$

$\cos 6x = \cos^2 3x - \sin^2 3x$

Example 4 - Deriving a Triple-Angle Formula

$$\sin 3x = \sin (2x + x)$$

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Example 5 - Reducing a Power

Rewrite $\sin^4 x$ as a sum of first powers of the cosines of multiple angles.

Half- Angle Formulas

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$$

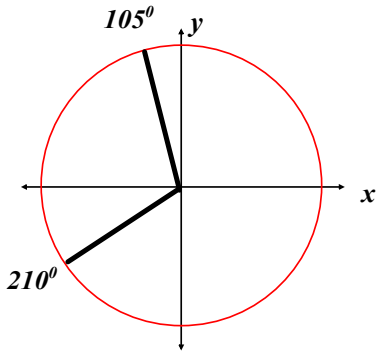
$$\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$$

$$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

The signs depend on what quadrant $u/2$ is in.

Example 6 - Using a Half-Angle Formula

Find the exact value of $\sin 105^0$



Example 7 - Solving a Trig Equation

Find all solutions in the interval $[\,0, 2\pi\,]$

$$2 - \sin^2 x = 2\cos^2 (x/2)$$

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}\left[\cos(u - v) - \cos(u + v)\right]$$

$$\cos u \cos v = \frac{1}{2}\left[\cos(u - v) + \cos(u + v)\right]$$

$$\sin u \cos v = \frac{1}{2}\left[\sin(u + v) + \sin(u - v)\right]$$

$$\cos u \sin v = \frac{1}{2}\left[\sin(u + v) - \sin(u - v)\right]$$

Example 8 - Writing Products as Sums

Rewrite the poduct as a sum or difference

$$\cos 5x \sin 4x$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$$

Example 9 - Using a Sum-to-Product Formula

Find the exact value of $\cos 195^0 + \cos 105^0$

Example 10- Solving a Trig Equation

Find all solutions in the interval $[\,0\,,\,2\pi\,]$

$$\sin 5x + \sin 3x = 0$$

Example 11 - Verifying a Trig Identity

Verify

$$\frac{\sin t + \sin 3t}{\cos t + \cos 3t} = \tan 2t$$