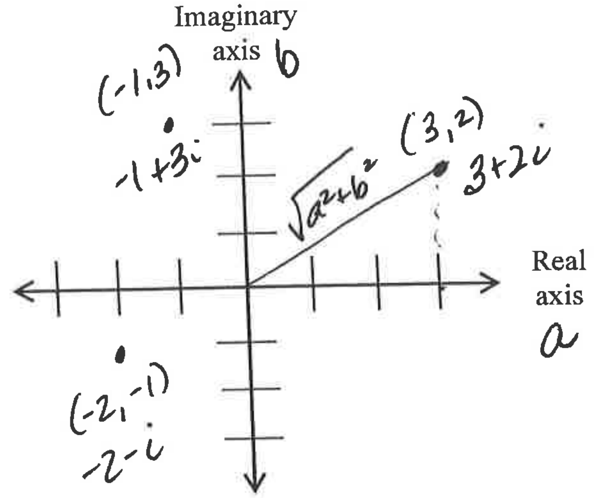
Complex Plane

$a+bi$



Absolute Value of a Complex Number

The absolute value of the complex number $z=a+bi$ is given by:

$$|a+bi|=\sqrt{a^2+b^2}$$

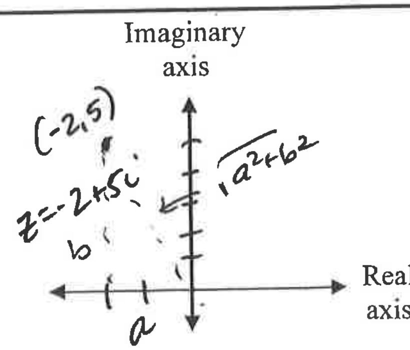
DIST FROM THE ORIGIN

What if $b=0$?

$$|a+0i|=\sqrt{a^2+0^2}$$
$$=\sqrt{a^2}$$
$$=|a|$$

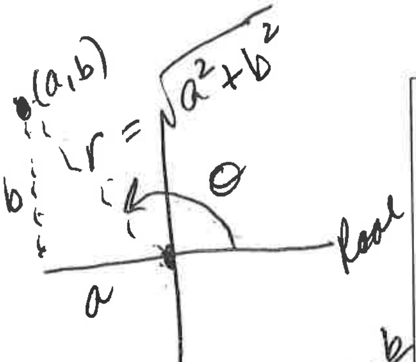
Example: Finding the Absolute Value of a Complex Number

Plot $z=-2+5i$ and find its absolute value.



$$|z|=\sqrt{(-2)^2+(5)^2}$$
$$\boxed{=\sqrt{29}}$$

Trig Form of a Complex Number



$\cos\theta = \frac{a}{r}$ $\sin\theta = \frac{b}{r}$

$r\cos\theta = a$ $r\sin\theta = b$

The trig form of a complex number $z = a + bi$ is given by:

$$\boxed{z = r(\cos\theta + i\sin\theta)}$$

where $a = r\cos\theta$, $b = r\sin\theta$, $r = \sqrt{a^2 + b^2}$ and

$$\tan\theta = \frac{b}{a}$$

The number r is the **modulus** of z, and θ is and **argument** of z

Example: Write the complex number $z = \overset{a}{-2} - \overset{b}{2i\sqrt{3}}$ in trigonometric form.

① Find r: $r = \sqrt{(-2)^2 + (-2\sqrt{3})^2} = \sqrt{4+12}$
$= \sqrt{16}$
$= 4$

② Find θ = $\tan\theta = \frac{b}{a}$

$\tan\theta = \frac{-2\sqrt{3}}{-2} = \sqrt{3}$ $\theta = \frac{\pi}{3} \Rightarrow \theta$ III

$\theta = \frac{4\pi}{3}$ $\boxed{z = 4\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)}$

Example: Write the complex number in standard form $a + bi$. $z = \sqrt{8}\left[\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right]$

$\sqrt{8}\left(\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right)$

$\sqrt{8}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

$2\sqrt{2}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

$\boxed{\sqrt{2} - \sqrt{6}\,i}$

Product and Quotient of Two Complex Numbers

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ be complex numbers

$$z_1 z_2 = r_1 r_2\left[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right]$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)\right]$$

Note: × multiply r add θ

÷ divide r subtract θ

Example: Find the product $z_1 z_2$ of the complex numbers: $z_1 = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$, $z_2 = 8\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$

$$z_1 z_2 = 16\left(\cos\left(\frac{2\pi}{3}+\frac{11\pi}{6}\right) + i\sin\left(\frac{2\pi}{3}+\frac{11\pi}{6}\right)\right)$$

$$= 16\left(\cos\frac{15\pi}{6} + i\sin\frac{15\pi}{6}\right)$$

$$= 16\left(\cos\frac{5\pi}{2} + i\sin\frac{5\pi}{2}\right)$$

$$= 16\left(0 + i(1)\right) = \boxed{16i}$$

Example: Find the quotient $\frac{z_1}{z_2}$ of the complex numbers: $z_1 = 24(\cos 300^0 + i\sin 300^0)$, $z_2 = 8(\cos 75^0 + i\sin 75^0)$

$$\frac{z_1}{z_2} = 3\left(\cos(300^\circ - 75^\circ) + i\sin(300^\circ - 75^\circ)\right)$$

$$= 3\left(\cos 225^\circ + i\sin 225^\circ\right)$$

$$= 3\left(-\frac{\sqrt{2}}{2} + i\left(-\frac{\sqrt{2}}{2}\right)\right)$$

$$\boxed{= \frac{-3\sqrt{2}}{2} - \frac{3i\sqrt{2}}{2}}$$

DeMoivre's Theorem

If $z = r(\cos\theta + i\sin\theta)$ is a complex number and n is a positive integer, then

$$z^n = [r(\cos\theta + i\sin\theta)]^n$$
$$= r^n(\cos n\theta + i\sin n\theta)$$

Example: Use DeMoivre's Theorem to find $(\overset{a}{-1} + \overset{b}{i\sqrt{3}})^{12}$

① Find r: $r = \sqrt{(-1)^2 + (\sqrt{3})^2}$

$r = \sqrt{4} = 2$

② Find θ: $\tan\theta = \frac{\sqrt{3}}{-1} = -\sqrt{3}$

$\theta = \frac{2\pi}{3}$

③ Rewrite $z = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

④ $z^{12} = \left[2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)\right]^{12}$

$2^{12}\left(\cos\left(12\cdot\frac{2\pi}{3}\right) + i\sin\left(12\cdot\frac{2\pi}{3}\right)\right]$

$4096(\cos 8\pi + i\sin 8\pi)$

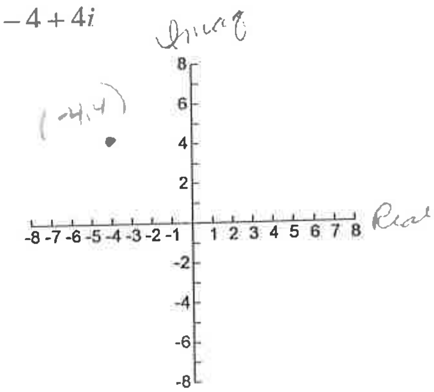
$4096(1 + i(0))$

4096

Plot the complex number and find its absolute value.

1) $-4+4i$

#5



$a=-4 \quad b=4$

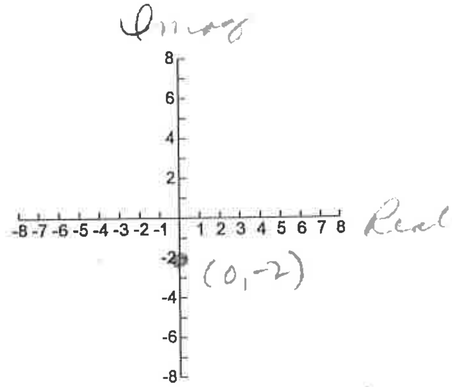
$|-4+4i| = \sqrt{(-4)^2+4^2}$

$= \sqrt{16+16}$

$= \sqrt{32}$

$\boxed{= 4\sqrt{2}}$

2) $-2i$



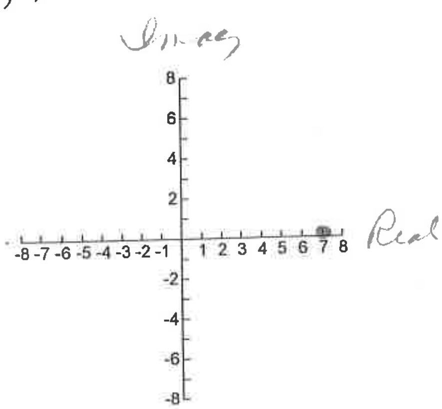
$a=0 \quad b=-2$

$|-2i| = \sqrt{0^2+(-2)^2}$

$= \sqrt{4}$

$\boxed{= 2}$

3) 7



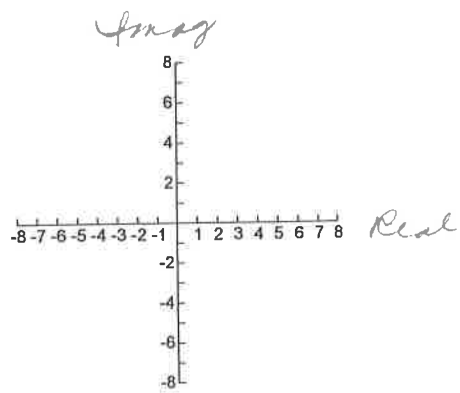
$a=7 \quad b=0$

$|7| = \sqrt{7^2+0^2}$

$= \sqrt{7^2}$

$\boxed{= 7}$

4) $2+i$



$a=2 \quad b=1$

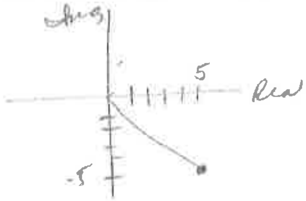
$|2+i| = \sqrt{2^2+1^2}$

$\boxed{= \sqrt{5}}$

Represent the complex number graphically, then find the trig form of the complex number.

(13) 5) $5-5i$ Q IV

$a=5 \quad b=-5$



(1) $r=\sqrt{5^2+(-5)^2}$

$=\sqrt{25+25}=\sqrt{50}=5\sqrt{2}$

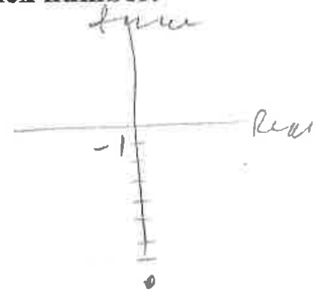
(2) $\theta \Rightarrow \tan\theta=\frac{-5}{5}=-1$

$\theta=\frac{7\pi}{4}$

$\boxed{z=5\sqrt{2}\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)}$

(14) 8) $-8i$

$a=0 \quad b=-8$



(1) $r=\sqrt{0^2+(-8)^2}$

$r=8$

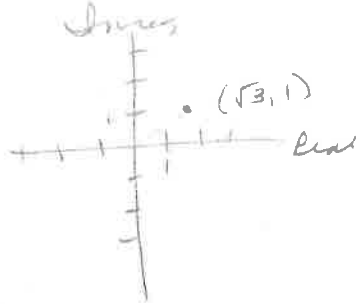
(2) $\theta \Rightarrow \tan\theta=\frac{-8}{0}$ undef.

$\theta=\frac{3\pi}{2}$

$\boxed{z=8\left(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\right)}$

(15) 7) $\sqrt{3}+i$

$a=\sqrt{3} \quad b=1$



(1) $r=\sqrt{(\sqrt{3})^2+1^2}$

$=\sqrt{3+1}=2$

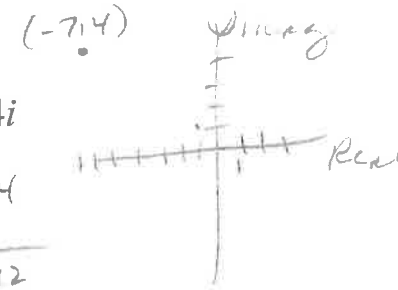
(2) $\theta \Rightarrow \tan\theta=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$

$\theta=\frac{\pi}{6}$

$\boxed{z=2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)}$

(21) 9) $-7+4i$

$a=-7 \quad b=4$



(1) $r=\sqrt{(-7)^2+4^2}$

$=\sqrt{49+16}=\sqrt{65}$

$-.519$

(2) $\theta \Rightarrow \tan\theta=\frac{4}{-7}$

$\theta=150.255^\circ$ or $\theta=2.622\,r$

$z=\sqrt{65}\left(\cos 150.255^\circ+i\sin 150.255^\circ\right)$

or $\sqrt{65}\left(\cos 2.622^r+i\sin 2.622^r\right)$

(17) 9) $-2(1+i\sqrt{3})$

$-2-2i\sqrt{3}$

(1) $r=\sqrt{(-2)^2+(-2\sqrt{3})^2}=4$

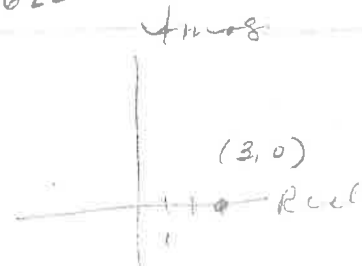
(2) $\theta \Rightarrow \tan\theta=\frac{-2\sqrt{3}}{-2}=\sqrt{3}$

Q III $\theta=\frac{4\pi}{3}$

$\boxed{z=4\left(\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}\right)}$

(23) 10) 3

$a=3 \quad b=0$



(1) $r=\sqrt{3^2+0^2}$

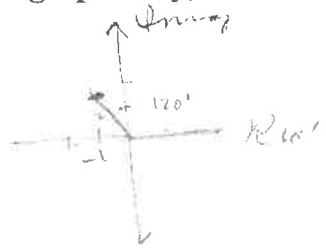
$r=3$

$\theta=0$

$\boxed{z=3\left(\cos 0^\circ+i\sin 0^\circ\right)}$

Represent the complex number graphically, then find the standard form of the number.

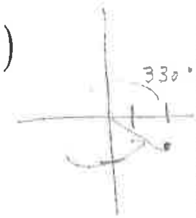
33 11) $2(\cos 120^0 + i\sin 120^0)$



$2\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$\boxed{-1 + i\sqrt{3}}$

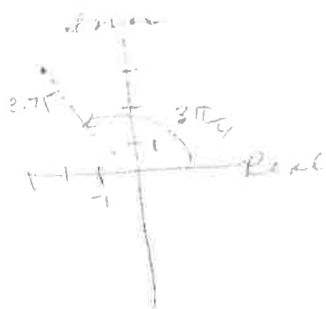
35 12) $\frac{3}{2}\left(\cos 330^0 + i\sin 330^0\right)$



$\frac{3}{2}\left(\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right)$

$\boxed{\frac{3\sqrt{3}}{4} - \frac{3i}{4}}$

37 13) $3.75\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$



$\frac{15}{4}\left(-\frac{\sqrt{2}}{2} + i\left(\frac{\sqrt{2}}{2}\right)\right)$

$\boxed{-\frac{15\sqrt{2}}{8} + \frac{15i\sqrt{2}}{8}}$

14) $6\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$6\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$\boxed{3 + 3i\sqrt{3}}$

5.19i

45 15) $5\left(\cos\frac{\pi}{9} + i\sin\frac{\pi}{9}\right)$

$5(.93969 + i(.34202))$

$\boxed{4.69845 + 1.7101i}$

48 16) $4\left(\cos 216.5^0 + i\sin 216.5^0\right)$

$4(-.80385 + i(-.59482))$

$\boxed{-3.2154 - 2.37929i}$

Perform the indicated operation and leave the result in trig form.

51 17) $\left[3\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right]\left[4\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\right]$

$12\left(\cos\left(\frac{\pi}{3}+\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{3}+\frac{\pi}{6}\right)\right)$

$\frac{2\pi}{6}$

$\boxed{12\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)}$

$12(0+i1)$

$\boxed{12i}$

53 18) $\left[\frac{5}{3}\left(\cos 140^0+i\sin 140^0\right)\right]\left[\frac{2}{3}\left(\cos 60^0+i\sin 60^0\right)\right]$

$\boxed{\frac{10}{9}\left(\cos 200^\circ+i\sin 200^\circ\right)}$

60 19) $\dfrac{\cos\left(\frac{7\pi}{4}\right)+i\sin\left(\frac{7\pi}{4}\right)}{\cos\pi+i\sin\pi}$

$\boxed{\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}}$

62 20) $\dfrac{9\left(\cos 20^0+i\sin 20^0\right)}{5\left(\cos 75^0+i\sin 75^0\right)}$

$\frac{9}{5}\left(\cos(-55)+i\sin(-55)\right)$

$\boxed{\frac{9}{5}\left(\cos 305^\circ+i\sin 305^\circ\right)}$

73 21) $(1+i)^3$ $a=1$ $b=1$

1. $r=\sqrt{2}$
2. $\theta \Rightarrow \frac{\pi}{4}$
3. $z=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

$z^3=\left[\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right]^3$

$\sqrt{8}$

$\boxed{=2\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)}$

79 22) $\left[5\left(\cos 20^0+i\sin 20^0\right)\right]^3$

$\boxed{125\left(\cos 60^\circ+i\sin 60^\circ\right)}$

$\frac{125}{2}+\frac{125\sqrt{3}}{2}i$