Geometric Sequence

Examples:

The nth Term of a Geometric Sequence

$$a_n = a_1 r^{n-1}$$

Examples:

- 1) Write the 1st five terms of the geometric sequence show first term is 3 and whose common ratio is 2
- 2) Find the 15th term of the geometric sequence whose first term is 20 and whose common ratio is 1.05.
- 3) Find the formula for the nth term of the following geometric sequence: 5, 15, 45, ...
- 4) The 4th term of a geometric sequence is 125 and the 10th term is 125/64. Find the 14th term. (Assume that the terms of the sequence are positive)

Geometric Series

Sum of a Finite Geometric Sequence

$$S_n = \sum_{i=1}^n a_i r^{i-1} = a_i \left(\frac{1 - r^n}{1 - r} \right)$$

Example: Find the sum $\sum_{n=1}^{12} 4(0.3)^n$

Sum of an Infinite Geometric Sequence

If |r| < 1 in a geometric sequence, then

$$S = \sum_{i=0}^{\infty} a_1 r^i = \frac{a_1}{1 - r}$$

Examples: Using your calculator, find the first 6 partial sums of the series. Then find the sum of the Series. Note: cumSum(seq(f(n), n, LL, UL)

Find the sum $3 + 0.3 + 0.03 + 0.003 + \dots$

Examples:

1) Determine whether or not the sequence is geometric. If yes, find the common ratio.

6,18,30,42,...

$$1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \dots$$
 $\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, \dots$

2) Write the first 5 terms of the geometric sequence :

 $a_1 = 6;$ r = 3

- 3) Find the 10th term of the geometric sequence: 5, 30, 180
- 4) Find the sum: $\sum_{i=1}^{7} 64 \left(-\frac{1}{2}\right)^{i-1}$
- 5) Find the sum: $\sum_{n=0}^{\infty} -3(0.9)^n$
- 6) Find the rational number representation of the repeating decimal: $0.3\overline{18}$

1) A principal of \$1000 is invested at 3% interest. Find the amount after 10 years is the interest is compounded a) annually, b) monthly

2) You go to work for a company that pays \$0.01 the first day, \$0.02 the second day, \$0.04 the third day, and so on. If the daily wage keeps doubling, what will your total income be after working a) 29 days, b) 30 days) c) 31 days?

3) Find the rational number representation of the repeating decimal: $0.\overline{297}$

4) A ball is dropped from a height of 16 feet. Each time it drops h feet, it rebounds 0.81 h feet. Find the total vertical distance traveled by the ball.

Geometric Sequence & Series

Date____

Determine if the sequence is geometric. If it is, find the common ratio, the term named in the problem, and the explicit formula.

1) $5, \frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \dots$ Find a_{12} 2) -4, -8, -16, -32, ... Find a_9

3) -1, 3, -9, 27, ... Find a_{12}

4) -96, -48, -24, -12, ... Find a_0

Given two terms in a geometric sequence find the explicit formula.

5)
$$a_5 = 512$$
 and $a_4 = -128$

6)
$$a_2 = 8$$
 and $a_3 = 32$

Given two terms in a geometric sequence find the common ratio and the 8th term.

7)
$$a_2 = -9$$
 and $a_6 = -729$

8)
$$a_6 = 4096$$
 and $a_3 = 64$

Given a term in a geometric sequence and the common ratio find the 8th term and the explicit formula.

9)
$$a_3 = 18, r = 3$$

10)
$$a_6 = -15552$$
, $r = -6$

Evaluate the related series of each sequence.

11) 4,
$$\frac{8}{3}$$
, $\frac{16}{9}$, $\frac{32}{27}$

12)
$$-1$$
, $\frac{1}{2}$, $-\frac{1}{4}$, $\frac{1}{8}$

Evaluate each geometric series described.

13)
$$1 + 2 + 4 + 8...$$
, $n = 7$

14)
$$1-5+25-125...$$
, $n=6$

15)
$$\sum_{i=1}^{8} 6^{i-1}$$

16)
$$\sum_{n=1}^{8} (-5)^{n-1}$$

Determine the number of terms n in each geometric series.

17)
$$a_1 = 3$$
, $r = -6$, $S_n = 119973$

18)
$$a_1 = 1$$
, $r = -6$, $S_n = -185$

Evaluate each infinite geometric series described.

19)
$$4+2+1+\frac{1}{2}$$
...

$$20) \ \frac{9375}{1024} - \frac{1875}{256} + \frac{375}{64} - \frac{75}{16} \dots$$

Determine the common ratio of the infinite geometric series.

21)
$$a_1 = -1.9$$
, $S = -1.1875$

22)
$$a_1 = 3.2$$
, $S = 4$