Arithmetic Sequence

Determine if the sequence is arithmetic. If it is, find the common difference.

1) $-27, -24, -21, -18, \ldots$ (+3, +3, +3)

Yes; $d = 3$

2) $17, 21, 25, 29, \ldots$ (+4, +4, +4)

Yes; $d = 4$

3) $18, 23, 28, 33, \ldots$ (+5, +5, +5)

Yes; $d = 5$

4) $-19, -39, -59, -79, \ldots$

Yes; $d = -20$

Determine if the sequence is arithmetic. If it is, find the common difference, the 52nd term, and the explicit formula.

5) $-19, -39, -59, -79, \ldots$ Yes; $d = -20$

$a_{52} = a_1 + (n-1)d$

$a_{52} = -19 + (52-1)(-20)$

$a_{52} = -19 + (51)(-20)$

$a_{52} = -1039$

$a_n = -19 + (n-1)(-20)$

$a_n = -19 - 20n + 20$

$a_n = -20n + 1$

6) $25, 55, 85, 115, \ldots$ Yes $d = 30$

$a_{52} = 25 + (52-1)(30)$

$a_{52} = 25 + (51)(30)$

$a_{52} = 1555$

$a_n = 25 + (n-1)30$

$a_n = 25 + 30n - 30$

$a_n = 30n - 5$

7) $25, 125, 225, 325, \ldots$ Yes $d = 100$

$a_{52} = a_1 + (52-1)(100)$

$a_{52} = 25 + 51(100)$

$a_{52} = 5125$

$a_n = 25 + (n-1)100$

$a_n = 25 + 100n - 100$

$a_n = 100n - 75$

8) $-12, -21, -30, -39, \ldots$ (-9, -9, -9) Yes $d = -9$

$a_{52} = -12 + (52-1)(-9)$

$a_{52} = -12 + 51(-9)$

$a_{52} = -471$

$a_n = -12 + (n-1)(-9)$

$a_n = -12 - 9n + 9$

$a_n = -3 - 9n$

Determine if the sequence is arithmetic. If it is, find the common difference, the term named in the problem, the explicit formula, and the three terms in the sequence after the last one given.

9) $-11, -13, -15, -17, \ldots$ (-2, -2, -2)

Find a_{25}

Yes, $d = -2$

$-19, -21, -23$

$a_{25} = -11 + (24)(-2)$

$a_{25} = -59$

$a_n = -11 + (n-1)(-2)$

$a_n = -11 - 2n + 2$

$a_n = -2n - 9$

10) $33, 30, 27, 24, \ldots$ (-3, -3, -3)

Find a_{32}

Yes $d = -3$

$21, 18, 15$

$a_{32} = 33 + (31)(-3)$

$a_{32} = -60$

$a_n = 33 + (n-1)(-3)$

$a_n = 33 - 3n + 3$

$a_n = 36 - 3n$

11) $33, 53, 73, 93, \ldots$

Find a_{36}

Yes $d = 20$

$113, 133, 153$

$a_{36} = 33 + 35(20)$

$a_{36} = 733$

$a_n = 33 + (n-1)20$

$a_n = 33 + 20n - 20$

$a_n = 20n + 13$

12) $-4, 196, 396, 596, \ldots$

Find a_{40}

Yes $d = 200$

$796, 996, 1196$

$a_{40} = -4 + (39)(200)$

$a_{40} = 7796$

$a_n = -4 + (n-1)200$

$a_n = -4 + 200n - 200$

$a_n = -204 + 200n$

Given a term in an arithmetic sequence and the common difference find the 52nd term and the explicit formula.

13) $a_{17} = -138,\ d = -8$

$a_{17} = a_1 + (17-1)(-8)$

$-138 = a_1 + 16(-8)$

$-10 = a_1$

$a_n = -10 + (n-1)(-8)$

$a_n = -10 - 8n + 8$

$\boxed{a_n = -8n - 2}$

$\boxed{a_{52} = -418}$

14) $a_{33} = 6366,\ d = 200$

$a_{33} = a_1 + (33-1)200$

$6366 = a_1 + 32(200)$

$-34 = a_1$

$a_n = -34 + (n-1)(200)$

$a_n = -34 + 200n - 200$

$\boxed{a_n = 200n - 234}$

$\boxed{a_{52} = 10166}$

15) $a_{21} = -5,\ d = -2$

$a_{21} = a_1 + (21-1)(-2)$

$-5 = a_1 + (20)(-2)$

$35 = a_1$

$a_n = 35 + (n-1)(-2)$

$a_n = 35 - 2n + 2$

$\boxed{a_n = -2n + 37}$ $\boxed{a_{52} = -67}$

16) $a_{30} = 892,\ d = 30$

$a_{30} = a_1 + (30-1)(30)$

$892 = a_1 + (29)(30)$

$22 = a_1$

$a_n = 22 + (n-1)(30)$

$a_n = 22 + 30n - 30$

$\boxed{a_n = 30n - 8}$ $\boxed{a_{52} = 1552}$

Given two terms in an arithmetic sequence find the common difference, the 52nd term, and the explicit formula.

17) $a_{18} = -1715$ and $a_{32} = -3115$

$a_{32} = a_{18} + (32-18)d$

$-3115 = -1715 + (14)d$

$-1400 = 14d$

$\boxed{-100 = d}$

$a_{18} = a_1 + (17)(-100)$

$-1715 = a_1 - 1700$

$-15 = a_1$

$a_{52} = a_{32} + (52-32)(-100)$

$\boxed{a_{52} = -3115 + 20(-100) = -5115}$

$a_n = -15 + (n-1)(-100)$

$a_n = -15 - 100n + 100$

$\boxed{a_n = -100n + 85}$

18) $a_{16} = 160$ and $a_{39} = 390$

$a_{39} = a_{16} + (39-16)d$

$390 = 160 + 23d$

$230 = 23d$

$\boxed{10 = d}$

$a_{16} = a_1 + (16-1)(10)$

$160 = a_1 + 15(10)$

$10 = a_1$

$a_n = 10 + (n-1)(10)$

$a_n = 10 + 10n - 10$

$\boxed{a_n = 10n}$

$a_{52} = 10(52)$

$\boxed{a_{52} = 520}$

19) $a_{19} = 1798$ and $a_{31} = 2998$

$a_{31} = a_{19} + (31-19)d$

$2998 = 1798 + (12)d$

$1200 = 12d$

$\boxed{100 = d}$

$a_{19} = a_1 + (19-1)100$

$1798 = a_1 + 18(100)$

$-2 = a_1$

$a_n = -2 + (n-1)100$

$a_n = -2 + 100n - 100$

$\boxed{a_{52} = 5098}$

20) $a_{16} = -1527$ and $a_{38} = -3727$

$a_{38} = a_{16} + (38-16)d$

$-3727 = -1527 + (22)d$

$-2200 = 22d$

$\boxed{-100 = d}$

$a_{16} = a_1 + (16-1)d$

$-1527 = a_1 + 15(-100)$

$-27 = a_1$

$a_n = a_1 + (n-1)d$

$a_n = -27 + (n-1)(-100)$

$a_n = -27 - 100n + 100$

$\boxed{a_n = -100n + 73}$ $\boxed{a_{52} = -5127}$

Arithmetic Series

Evaluate the related series of each sequence.

1) 35, 45, 55, 65 $35+45+55+65$

200

2) 1, 3, 5, 7 $1+3+5+7$

16

3) $-\frac{19}{4}, -\frac{25}{4}, -\frac{31}{4}, -\frac{37}{4}, -\frac{43}{4}, -\frac{49}{4}, -\frac{55}{4}$ $\overset{-\frac{3}{2}}{}$ $n=7$

-64.75 or $-\frac{259}{4}$ ✓

4) $\frac{10}{3}, 4, \frac{14}{3}, \frac{16}{3}, 6, \frac{20}{3}, \frac{22}{3}$ $\overset{+\frac{2}{3}}{}$

$37.\overline{3}$ or $\frac{112}{3}$

Evaluate each arithmetic series described.

5) $\sum_{k=1}^{12}\left(-2+\frac{1}{2}k\right)$ $a_1 = -\frac{3}{2}$, $a_{12} = 4$

$S_{12} = \frac{12}{2}\left(-\frac{3}{2}+4\right)$

15 ✓

6) $\sum_{n=1}^{10}(4n-13)$

$S_{10} = \frac{10}{2}(-9+27)$

90 ✓

7) $\sum_{n=1}^{8}(6n-15)$ $a_1 = -9$, $a_8 = 33$

$S_8 = \frac{8}{2}(-9+33)$

$= 96$ ✓

8) $\sum_{n=1}^{40}(2.7n+0.6)$ $a_1 = 3.3$, $a_{40} = 108.6$

$S_{40} = \frac{40}{2}(3.3+108.6)$

$= 2238$ ✓

9) $a_1 = 19,\ a_n = 117,\ n = 15$

$S_{15} = \frac{15}{2}(19+117)$

$= 1020$ ✓

10) $a_1 = 25,\ a_n = 77,\ n = 14$

$S_{14} = \frac{14}{2}(25+77)$

$= 714$

11) $a_1 = 28,\ d = 10,\ n = 13$

$S_{13} = \frac{13}{2}(28+148)$

$= 1144$ ✓

$a_{13} = a_1 + (13-1)10$

$a_{13} = 28 + 12(10)$

$a_{13} = 148$

12) $a_1 = 35,\ d = 9,\ n = 13$

$S_{13} = \frac{13}{2}(35+143)$

$= 1157$ ✓

$a_{13} = a_1 + (13-1)(9)$

$a_{13} = 35 + 12(9)$

$a_{13} = 143$

13) $11 + 15 + 19 + 23\ldots,\ n = 9$ $d = 4$

$S_9 = \frac{9}{2}(11 + 43)$

$= \boxed{243}$ ✓

$a_9 = a_1 + (9-1)(4)$

$a_9 = 11 + 8(4)$

$a_9 = 43$

14) $4 + 7 + 10 + 13\ldots,\ n = 12$ $d = 3$

$S_{12} = \frac{12}{2}(4 + 37)$

$= \boxed{246}$ ✓

$a_{12} = 4 + (12-1)(3)$

$a_{12} = 4 + 33$

$a_{12} = 37$

Determine the number of terms *n* in each arithmetic series.

15) $a_1 = 9,\ a_n = 24,\ S_n = 99$

$99 = \frac{n}{2}(9 + 24)$

$99 = \frac{n}{2}(33)$

$3 = \frac{n}{2}$

$\boxed{6 = n}$ ✓

16) $a_1 = 19,\ a_n = 264,\ S_n = 7075$

$7075 = \frac{n}{2}(19 + 264)$

$7075 = \frac{n}{2}(283)$

$25 = \frac{n}{2}$

$\boxed{50 = n}$ ✓

17) $a_1 = 12,\ d = 8,\ S_n = 672$ $a_n = 12 + (n-1)8$, $12 + 8n - 8$, $8n + 4$

$672 = \frac{n}{2}(12 + a_n)$

$672 = \frac{n}{2}(12 + 8n + 4)$

$672 = \frac{n}{2}(16 + 8n)$

$672 = 4n^2 + 8n$

$168 = n^2 + 2n$

$0 = n^2 + 2n - 168$

$\boxed{n = 12}$ ✓

18) $a_1 = -14,\ d = -8,\ S_n = -336$ $a_n = -14 + (n-1)(-8)$, $a_n = -8n - 6$

$-336 = \frac{n}{2}(-14 + a_n)$

$-336 = \frac{n}{2}(-14 - 8n - 6)$

$-336 = \frac{n}{2}(-20 - 8n)$

$0 = -4n^2 - 10n + 336$

$\boxed{n = 8}$ ✓

19) $\sum_{i=1}^{n}(-4i - 3) = -133$

$-133 = \frac{n}{2}(-7 + (-4n - 3))$

$-133 = \frac{n}{2}(-4n - 10)$

$-133 = -2n^2 - 5n$

$0 = -2n^2 - 5n + 133$

$0 = 2n^2 + 5n - 133$

$0 = 2n^2 - 14n + 19n - 133$

$0 = 2n(n-7) + 19(n-7)$

$(2n+19)(n-7) = 0$

$n = \frac{-19}{2}$ $\boxed{n = 7}$ ✓

20) $\sum_{m=1}^{n}(1 - 6m) = -705$

$-705 = \frac{n}{2}(-5 + (1 - 6n))$

$-705 = \frac{n}{2}(-6n - 4)$

$-705 = -3n^2 - 2n$

$0 = -3n^2 - 2n + 705$

$0 = -3n^2 + 45n - 47n + 705$

$0 = -3n(n-15) - 47(n-15)$

$-3n - 47 = 0$ $n - 15 = 0$

$n = \frac{47}{3}$ $\boxed{n = 15}$ ✓

HPC Name Key ID: 1

Arithmetic Sequence & Series

Date

Determine if the sequence is arithmetic. If it is, find the common difference, the term named in the problem, and the explicit formula.

1) 37, 43, 49, 55, ... $d = 6$
Find a_{25}

$a_n = 37 + (n-1)(6)$
$a_n = 37 + 6n - 6$
$a_n = 31 + 6n$
$a_{25} = 31 + 6(25)$
$a_{25} = 181$

2) 37, 44, 51, 58, ... $d = 7$
Find a_{30}

$a_n = 37 + (n-1)(7)$
$a_n = 37 + 7n - 7$
$a_n = 30 + 7n$
$a_{25} = 30 + 7(30)$
$a_{30} = 240$

3) 0, 9, 18, 27, ... $d = 9$
Find a_{39}

$a_n = 0 + (n-1)(9)$
$a_n = 9n - 9$
$a_{39} = 0 + (39-1)(9)$
$a_{39} = 342$

4) $\frac{3}{10}, -\frac{1}{30}, -\frac{11}{30}, -\frac{7}{10}, \ldots$ $d = -\frac{1}{3}$
Find a_{36}

$a_n = \frac{3}{10} + (n-1)(-\frac{1}{3})$
$a_n = \frac{3}{10} - \frac{1}{3}n + \frac{1}{3}$
$a_n = -\frac{1}{3}n + \frac{19}{30}$
$a_{36} = -\frac{1}{3}(36) + \frac{19}{30}$
$a_{36} = -\frac{341}{30}$

Given two terms in an arithmetic sequence find the explicit formula.

5) $a_{13} = 7.1$ and $a_{36} = 20.9$

$a_{36} = a_{13} + (36-13)d$
$20.9 = 7.1 + 23d$
$13.8 = 23d$
$.6 = d$

$a_{13} = a_1 + (13-1)(.6)$
$7.1 = a_1 + 12(.6)$
$-.1 = a_1$

$a_n = -.1 + (n-1)(.6)$
$a_n = .6n - .7$ ✓

6) $a_{15} = \frac{71}{3}$ and $a_{39} = \frac{191}{3}$

$a_{39} = a_{15} + (39-15)d$
$\frac{191}{3} = \frac{71}{3} + 24d$
$\frac{120}{3} = 24d$
$40 = 24d$
$\frac{5}{3} = d$

$a_{15} = a_1 + 14(\frac{5}{3})$
$\frac{71}{3} = a_1 + \frac{70}{3}$
$\frac{1}{3} = a_1$

$a_n = \frac{1}{3} + (n-1)(\frac{5}{3})$
$a_n = \frac{1}{3} + \frac{5}{3}n - \frac{5}{3}$
$a_n = \frac{5}{3}n - \frac{4}{3}$ ✓

Given two terms in an arithmetic sequence find the common difference, the term named in the problem, and the explicit formula.

7) $a_{20} = -\frac{93}{10}$ and $a_{36} = -\frac{173}{10}$

Find a_{31}

$a_{36} = a_{20} + (36-20)d$

$-\frac{173}{10} = -\frac{93}{10} + 16d$

$-8 = 16d$

$-\frac{1}{2} = d$

$a_{20} = a_1 + 19(-\frac{1}{2})$

$-\frac{93}{10} = a_1 + -\frac{19}{2}$

$\frac{1}{5} = a_1$

$a_n = \frac{1}{5} + (n-1)(-\frac{1}{2})$

$a_n = \frac{1}{5} - \frac{1}{2}n + \frac{1}{2}$

$a_n = \frac{7}{10} - \frac{1}{2}n$

$a_{31} = \frac{7}{10} - \frac{1}{2}(31) = -\frac{74}{5}$

8) $a_{13} = -6$ and $a_{37} = -22$

Find a_{32}

$a_{37} = a_{13} + (37-13)d$

$-22 = -6 + 24d$

$-16 = 24d$

$-\frac{2}{3} = d$

$a_{13} = a_1 + 12(-\frac{2}{3})$

$-6 = a_1 + (-8)$

$2 = a_1$

$a_n = 2 + (n-1)(-\frac{2}{3})$

$a_n = 2 - \frac{2}{3}n + \frac{2}{3}$

$a_n = -\frac{2}{3}n + \frac{8}{3}$

$a_{32} = -\frac{2}{3}(32) + \frac{8}{3}$

$a_{32} = -\frac{56}{3}$

Given two terms in an arithmetic sequence find the explicit formula.

9) $a_{19} = -\frac{19}{4}$ and $a_{37} = -\frac{43}{4}$

$a_{37} = a_{19} + (37-19)d$

$-\frac{43}{4} = -\frac{19}{4} + 18d$

$-6 = 18d$

$-\frac{1}{3} = d$

$a_{19} = a_1 + 18(-\frac{1}{3})$

$-\frac{19}{4} = a_1 - 6$

$\frac{5}{4} = a_1$

$a_n = \frac{5}{4} + (n-1)(-\frac{1}{3})$

$a_n = -\frac{1}{3}n + \frac{19}{12}$

10) $a_{17} = \frac{77}{3}$ and $a_{30} = \frac{142}{3}$

$a_{30} = a_{17} + (30-17)d$

$\frac{142}{3} = \frac{77}{3} + (13)d$

$\frac{65}{3} = 13d$

$\frac{5}{3} = d$

$a_{17} = a_1 + (16)(\frac{5}{3})$

$\frac{77}{3} = a_1 + \frac{80}{3}$

$-1 = a_1$

$a_n = -1 + (n-1)\frac{5}{3}$

$a_n = +\frac{5}{3}n - \frac{8}{3}$

Evaluate the related series of each sequence.

11) 13,+23,+33,+ 43

112

12) 0,+−2,+−4,+−6,+−8,+−10,+−12

−42

Evaluate each arithmetic series described.

13) $\sum_{n=1}^{11}(4n-11)$

$S_{11} = \frac{11}{2}(-7+33)$

$S_{11} = 143$ ✓

14) $\sum_{k=1}^{10}(10k-20)$

$S_{10} = \frac{10}{2}(-10+80)$

$S_{10} = 350$ ✓

15) $\sum_{m=5}^{13}(10m-11)$ $n = 13-5+1 = 9$

$S = \frac{9}{2}(39+119)$

$= 711$ ✓

16) $\sum_{m=5}^{10}(2m+3)$ $n = 10-5+1$

$S = \frac{6}{2}(13+23)$

$= 108$ ✓

17) $9+13+17+21...,\ n=17$ $d=4$

$S_{17} = \frac{17}{2}(9+73)$

$= 697$ ✓

$a_{17} = 9+(17-1)(4)$

$a_{17} = 73$

18) $12+15+18+21...,\ n=16$ $d=3$

$S_{16} = \frac{16}{2}(12+57)$

$= 552$ ✓

$a_{16} = 12+(16-1)(3)$

$= 57$

1) A brick patio has the approximate shape of a trapezoid. The patio has 18 rows of bricks. The first row has 14 bricks and the 18th row has 31 bricks. How many bricks are in the patio?

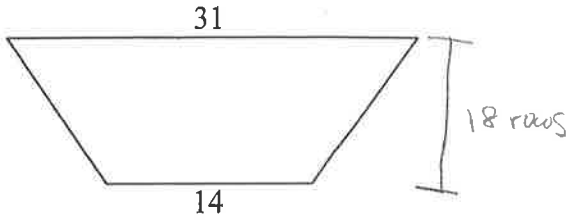
$a_1 = 14$

$a_{18} = 31$

Find S_{18}

$S_{18} = \frac{18}{2}(14+31)$

$= 405$ Bricks



2) Each row in a small auditorium has two more seats than the preceding row. Find the seating capacity if the front row seats 25 people and there are 15 rows of seats.

$a_1 = 25$

$n = 15$

$d = 2$

$a_{15} =$

$a_{15} = 25 + 14(2)$

$a_{15} = 53$

$S_{15} = \frac{15}{2}(25+53)$

$= 585$ seats

3) A small hardware store makes a profit of $20,000 during its first year. The store owner sets a goal of raising profits by $5000 each year for 4 years. Assuming that this goal is met, find the total profit during the first 5 years of business.

$a_1 = 20,000$

$d = 5000$

$a_5 = 20,000 + 5000(4)$

$a_5 = 40,000$

$S_5 = \frac{5}{2}(20,000 + 40,000)$

$= \$150,000$

4) Consider a job offer with a starting salary of $32,500 with an annual raise of $1500.

a) What is the salary in the 6th year?

$a_1 = 32,500$

$d = 1,500$

BEG 6th yr $\rightarrow a_6 = 32,500 + 5(1500)$

$a_6 = 40,000$

$\$40,000$

b) What is the total compensation for the company through 6 full years of employment/?

$S_6 = \frac{6}{2}(32,500 + 40,000)$

$= 217,500$

$\$217,500$