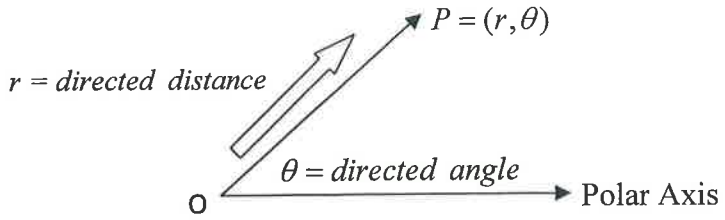
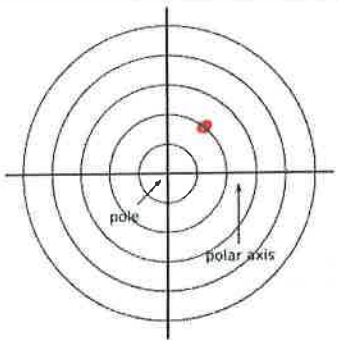
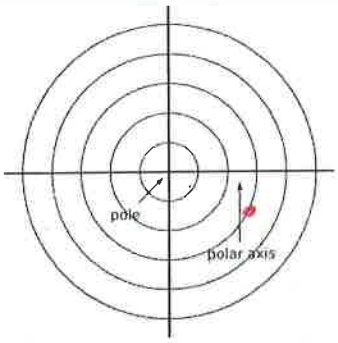
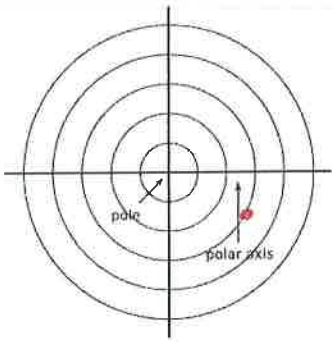


Plotting Points in the Polar Coordinate System

a) $(r,\theta) = \left(2, \frac{\pi}{3}\right)$ b) $(r,\theta) = \left(3, -\frac{\pi}{6}\right)$ c) $(r,\theta) = \left(3, \frac{11\pi}{6}\right)$







Rectangular Coordinates (x, y)

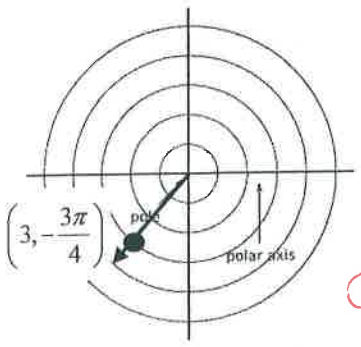
Polar Coordinates (r, θ)

Multiple Representations of Points

$(r,\theta) = (r, \theta \pm 2\pi n)$

$= (-r, \theta \pm (2n+1)\pi)$

Find 3 Additional Polar Representations $-2\pi < \theta < 2\pi$



① $\left(3, -\frac{3\pi}{4}\right)$

② $\left(3, -\frac{3\pi}{4} + 2\pi\right) \rightarrow \left(3, \frac{5\pi}{4}\right)$

③ $\left(-3, \frac{\pi}{4}\right)$

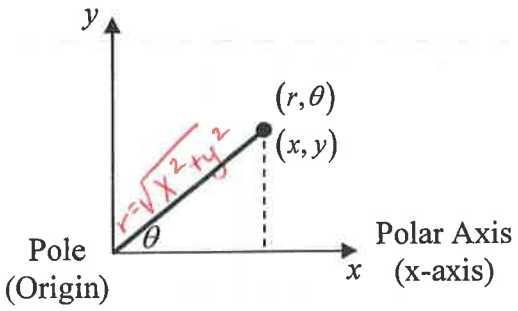
④ $\left(-3, -\frac{7\pi}{4}\right)$

Coordinate Conversion

The polar coordinates (r,θ) are related to the rectangular coordinates (x,y) as follows:

$$x = r\cos\theta \qquad \tan\theta = \frac{y}{x}$$

$$y = r\sin\theta \qquad r^2 = x^2 + y^2$$



Polar to Rectangular Conversion

Convert each polar coordinate to rectangular coordinates.

a) $(2,\pi)$ $(r,\theta) \rightarrow (x,y)$ $(-2,0)$

$x = r\cos\theta$ $y = r\sin\theta$

$x = 2\cos\pi$ $y = 2\sin\pi$

$x = 2(-1)$ $y = 2(0)$

$x = -2$ $y = 0$ $(-2,0)$

b) $\left(\sqrt{3},\frac{\pi}{6}\right)$ $(r,\theta) \rightarrow (x,y)$

$x = r\cos\theta$ $y = r\sin\theta$

$x = \sqrt{3}\cos\frac{\pi}{6}$ $y = \sqrt{3}\sin\frac{\pi}{6}$

$x = \sqrt{3}\left(\frac{\sqrt{3}}{2}\right)$ $y = \sqrt{3}\left(\frac{1}{2}\right)$

$x = \frac{3}{2}$ $y = \frac{\sqrt{3}}{2}$ $\left(\frac{3}{2},\frac{\sqrt{3}}{2}\right)$

Rectangular to Polar Conversion

Convert each rectangular coordinate to polar coordinates.

a) $(-1,1)$ $(x,y) \rightarrow (r,\theta)$ QII

$r = \sqrt{(-1)^2+1^2}$ $\tan\theta = \frac{y}{x}$

$r = \sqrt{2}$ $\tan\theta = \frac{1}{-1} = -1$

$\theta = \frac{3\pi}{4}$

$\left(\sqrt{2},\frac{3\pi}{4}\right)$

b) $(0,2)$ $(x,y) \rightarrow (r,\theta)$

$r = \sqrt{0^2+2^2}$ $\tan\theta = \frac{2}{0} \rightarrow$ DNE

$r = \sqrt{4}$ $\theta = \frac{\pi}{2}$

$r = 2$

$\left(2,\frac{\pi}{2}\right)$

Equation Conversion

Point Conversion : Polar to Rectangular → Straightforward
Rectangular to Polar → More Involved

Equation Conversion: Polar to Rectangular → More Involved
Rectangular to Polar → Straightforward

Rectangular to Polar Equation – Replace x w/ $r\cos\theta$; y w/ $r\sin\theta$

Example: $y = x^2$

$r\sin\theta = (r\cos\theta)^2$

$r\sin\theta = r^2\cos^2\theta$

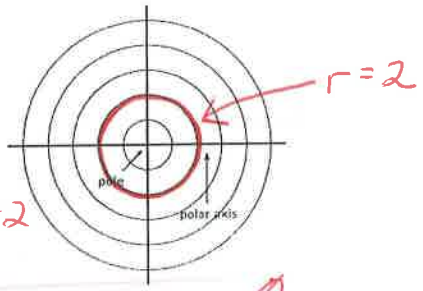
$\frac{\sin\theta}{\cos^2\theta} = r$

$\frac{1}{\cos\theta} \cdot \frac{\sin\theta}{\cos\theta} = r$

$\sec\theta\tan\theta = r$

Polar to Rectangular - Need r^2; $r\cos$; $r\sin$ or $\tan\theta$

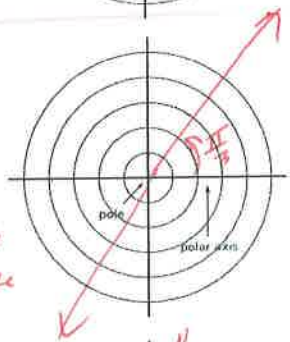
a) $r = 2$ ⟹ all Points 2 units from pole.



$r^2 = 4$

$x^2 + y^2 = 4$ ← Equation of ⊙ Centered @ origin w/ r=2

b) $\theta = \frac{\pi}{3}$ ⟹ all points that form angle of $\frac{\pi}{3}$ w/ + x-axis



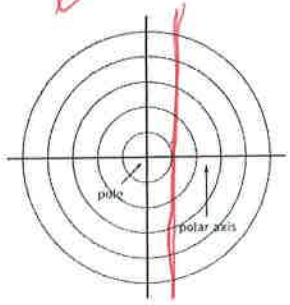
$\tan\theta = \frac{y}{x}$

$\tan\frac{\pi}{3} = \frac{y}{x}$

$\sqrt{3} = \frac{y}{x}$

$\sqrt{3}x = y$ Equation of line w/ slope of $\sqrt{3}$

c) $r = \sec\theta$ use $r\cos\theta = x$



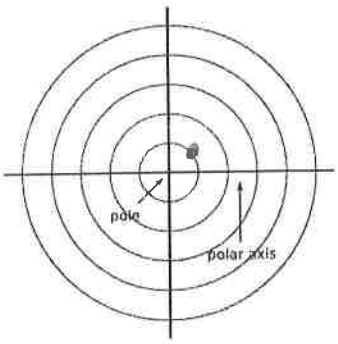
$r = \frac{1}{\cos\theta}$

$r\cos\theta = 1$

$x = 1$

Plot the polar coordinate. Convert the polar coordinate to rectangular.

1) $\left(-1, \frac{5\pi}{4}\right)$



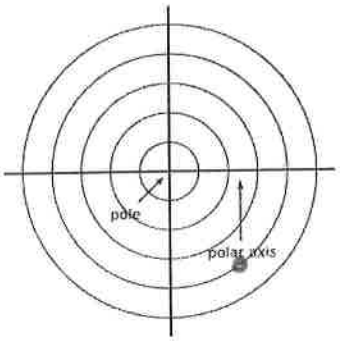
$r = -1$
$\theta = \frac{5\pi}{4}$

$x = (-1)\cos\frac{5\pi}{4}$
$x = -1\left(-\frac{\sqrt{2}}{2}\right)$
$x = \frac{\sqrt{2}}{2}$

$y = (-1)\sin\frac{5\pi}{4}$
$y = (-1)\left(-\frac{\sqrt{2}}{2}\right)$
$y = \frac{\sqrt{2}}{2}$

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

2) $\left(4, -\frac{\pi}{3}\right)$



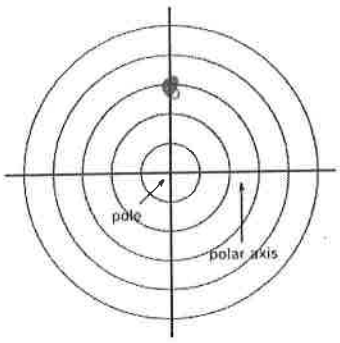
$r = 4$
$\theta = -\frac{\pi}{3}$

$x = 4\cos\left(-\frac{\pi}{3}\right)$
$x = 4\left(\frac{1}{2}\right)$
$x = 2$

$y = 4\sin\left(-\frac{\pi}{3}\right)$
$y = 4\left(-\frac{\sqrt{3}}{2}\right)$
$y = -2\sqrt{3}$

$(2, -2\sqrt{3})$

3) $(-3, -1.57)$ radians



$r = -3$
$\theta = 1.57$

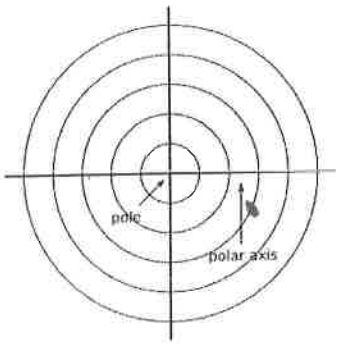
$x = -3\cos(-1.57)$
$x = -.0023889$

$y = -3\sin(-1.57)$
$y = 2.9999$

$(-.0024, 3)$

Plot the polar coordinate. Find three additional polar representations of the point, using $-2\pi < \theta < 2\pi$

4) $\left(-3, -\frac{7\pi}{6}\right)$



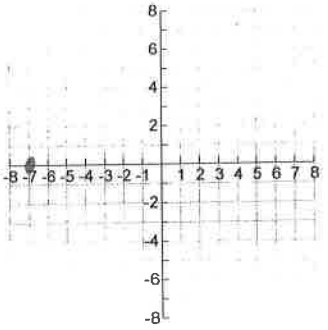
$\left(-3, \frac{5\pi}{6}\right)$

$\left(3, -\frac{\pi}{6}\right)$

$\left(3, \frac{11\pi}{6}\right)$

Plot the given rectangular coordinate and find TWO sets of polar coordinates for $0 \le \theta < 2\pi$

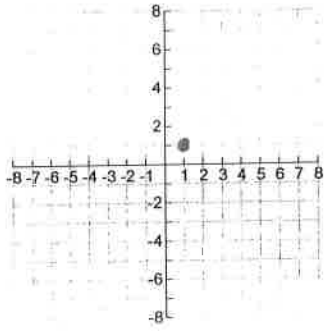
5) $(-7,0)$



$(7, \pi)$ $(-7, 0)$

$(7, -\pi)$

6) $(1,1)$



$r = \sqrt{1^2 + 1^2}$

$r = \sqrt{2}$

$\theta \Rightarrow \tan\theta = \frac{y}{x}$

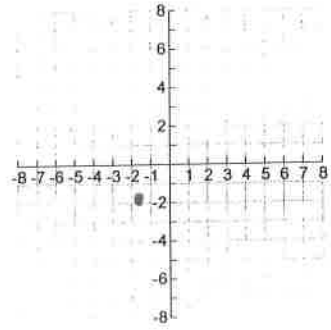
$\tan\theta = 1$

$\theta = \frac{\pi}{4}$

$(\sqrt{2}, \frac{\pi}{4})$ $(-\sqrt{2}, \frac{-3\pi}{4})$

$(\sqrt{2}, \frac{7\pi}{4})$ $(-\sqrt{2}, \frac{5\pi}{4})$

7) $(-\sqrt{3}, -\sqrt{3})$



$r = \sqrt{(-\sqrt{3})^2 + (-\sqrt{3})^2}$

$r = \sqrt{3+3}$

$r = \sqrt{6}$

$\tan\theta = \frac{-\sqrt{3}}{-\sqrt{3}} = 1$

$\theta = \frac{5\pi}{4}$

$(\sqrt{6}, \frac{5\pi}{4})$ $(-\sqrt{6}, \frac{\pi}{4})$

$(\sqrt{6}, -\frac{3\pi}{4})$ $(-\sqrt{6}, -\frac{7\pi}{4})$

Convert the rectangular equation to polar form. Assume $a > 0$.

8) $x^2 + y^2 = 81$

$r^2 = 81$

$r = 9$

9) $y = 4$

$r\sin\theta = 4$

$r = \frac{4}{\sin\theta}$

$r = 4\csc\theta$

10) $3x - 6y + 2 = 0$

$3r\cos\theta - 6r\sin\theta + 2 = 0$

$3r\cos\theta - 6r\sin\theta = -2$

$r(3\cos\theta - 6\sin\theta) = -2$

$r = \frac{-2}{3\cos\theta - 6\sin\theta}$

11) $xy = 4$

$(r\cos\theta)(r\sin\theta) = 4$

$r^2\cos\theta\sin\theta = 4$

$r^2(2\cos\theta\sin\theta) = 8$

$r^2\sin 2\theta = 8$

$r^2 = \frac{8}{\sin 2\theta}$

$r^2 = 8\csc 2\theta$

12) $(x^2 + y^2)^2 = 9(x^2 - y^2)$

$(r^2)^2 = 9((r\cos\theta)^2 - (r\sin\theta)^2)$

$r^4 = 9(r^2\cos^2\theta - r^2\sin^2\theta)$

$r^4 = 9r^2(\cos^2\theta - \sin^2\theta)$

$r^2 = 9(\cos^2\theta - \sin^2\theta)$

$r^2 = 9\cos 2\theta$

13) $y^2 = x^3$

$(r\sin\theta)^2 = (r\cos\theta)^3$

$r^2\sin^2\theta = r^3\cos^3\theta$

$\sin^2\theta = r\cos^3\theta$

$\frac{\sin^2\theta}{\cos^3\theta} = r$

$\tan^2\theta\sec\theta = r$

Convert the polar equations to rectangular form.

14) $r = 6\sin\theta$

$r^2 = 6r\sin\theta$

$x^2 + y^2 = 6y$

$x^2 - 6y + y^2 = 0$

15) $\theta = \frac{4\pi}{3}$

$\tan\theta = \frac{y}{x}$

$\tan\frac{4\pi}{3} = \frac{y}{x}$

$\sqrt{3} = \frac{y}{x}$

$\sqrt{3}\,x = y$

16) $r = 4$

$r^2 = 16$

$x^2 + y^2 = 16$

17) $r = -3\csc\theta$

$r = \frac{-3}{\sin\theta}$

$r\sin\theta = -3$

$y = -3$