6th Edition

HONORS PRECALCULUS
Intro to Calculus
Part 1

#1	11.1	#s 9, 18, 33-40, 46, 47
	11.2	#s 5, 9, 53
#2	LIMIT W.S.	#s 1-23 all
#3	PACKET p.5	#s 1-12 all
#4	PACKET p. 8	#s 1-12 all
#5	PACKET p.11	#s 1-12 all
#6	PACKET p.13	#s 1-9 all
#7	PACKET p.14	#s 1-18 all
#8	REVIEW W.S. p. 15-18	#s 1-18 all

- **Quiz on Limits; Average Rate of Change; Instantaneous Rate of Change**
- **Quiz on Derivatives**
- **Test on Packet Calc 1 Unit**

Solutions to Problems in the Packet - Packet pp.19-21

All work is to be done on loose leaf and/or graph paper to be handed in
Work on WS's may be done on the worksheet and kept in the packet

Rate of Change

Distance with Respect to Time

Suppose a rocket starts from rest at a point *O* and is at a distance *s* feet from *O* along a straight path at the end of *t* seconds. If the distance *s* feet at the time *t* seconds is given by the equation $s = 10t^2$, a function with domain $\Re$ and range the set of nonnegative real numbers is found. Of course, the physical situation would determine the meaningful domain and range of the function.

Using $s(t) = 10t^2$ to denote the function where $s(t)$ is the distance traveled in time *t* from *t* = 0.

$s(1) = 10(1)^2 = 10$ 10 ft is the distance traveled from *t* = 0 to *t* = 1.

$s(2) = 10(2)^2 = 40$ 40 ft is the distance traveled from *t* = 0 to *t* = 2.

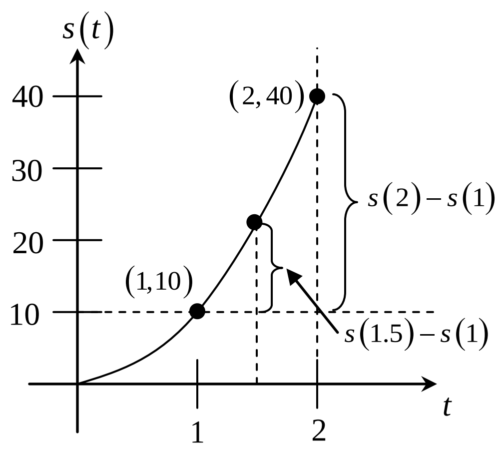
The average velocity over the time interval from *t* = 1 to *t* = 2 is: $\dfrac{\text{change in distance}}{\text{change in time}}$

$$\frac{s(2)-s(1)}{2-1} = \frac{40-10}{1} = 30$$

The average velocity over this interval is 30 ft/s. In the same way, the average velocity from *t* = 1 to *t* = 1.5 can be determined by:

$$\frac{s(1.5)-s(1)}{1.5-1} = \frac{10(1.5)^2-10(1)^2}{0.5} = \frac{22.5-10}{0.5} = 25$$

The average velocity over this interval is 25 ft/s. The figure below gives a graphical representation.



What is the velocity at $t = 1$? An answer for this question could be obtained by tabulating average velocities over short time intervals from $t = 1$ to $t = 1 + h$ for smaller and smaller positive values of *h*.

$$\frac{s(1+h)-s(1)}{(1+h)-1} = \frac{s(1+h)-s(1)}{h}$$

h	0.2	0.1	0.05	0.01	0.001
$\frac{s(1+h)-s(1)}{h}$	22	21	20.5	20.1	20.01

The chart above illustrates the ratio formed to calculate the average velocity over the interval $(1, 1+h)$.
It seems clear that the average velocity over the interval from $t = 1$ to $t = 1 + h$ is very close to 20 ft/s for small positive values of *h*. The ratio can be made as close to the desired 20 by taking *h* sufficiently small, as long as $h \neq 0$.

Definition: Limit	As the variable *x* approaches a certain number *a*, the function $f(x)$ will approach some number *L*. Another meaning of limit is the instantaneous velocity of a function. $$\lim_{x \to a} f(x) = L$$

EXAMPLE 1 Taking the Limit of a function

Take the limit of the following function

$$\lim_{x \to 4} \frac{x^2 - 7x + 12}{x - 4}$$ Original limit

$$\lim_{x \to 4} \frac{\cancel{(x-4)}(x-3)}{\cancel{x-4}} = x - 3$$ Factor completely and simplify

$4 - 3 = 1$ Apply the limit to *x* and solve

EXAMPLE 2 Finding average velocity and instantaneous velocity

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The distance of a rocket with respect to time can be modeled by the function $s(t)=10t^2$ measured in feet per second. Find the average velocity of a rocket from $t=1$ to $t=1.5$ then find the instantaneous velocity when $t=5$.

<u>SOLUTION</u>

To calculate the average velocity, we must evaluate the given time intervals into the following:

$\frac{s(t+h)-s(t)}{h}$, where $t+h=1.5, t=1,$ and $h=.5$

$$\frac{s(1.5)-s(1)}{.5}$$
$$=\frac{10(1.5)^2-10(1)}{.5}$$
$$=\frac{22.5-10}{.5}=\frac{12.5}{.5}=25$$

So the average velocity of the rocket over the time period 1 to 1.5 seconds is 25 ft/s.

To calculate the instantaneous velocity when $t=5$, we must take the limit of $\frac{s(t+h)-s(t)}{h}$ as h approaches zero.

$$\lim_{h\to 0}\frac{s(5+h)-s(5)}{h}$$
$$=\lim_{h\to 0}\frac{10(5+h)^2-10(5)^2}{h}$$
$$=\lim_{h\to 0}\frac{10\left(25+10h+h^2\right)-10(25)}{h}$$
$$=\lim_{h\to 0}\frac{250+100h+10h^2-250}{h}$$
$$=\lim_{h\to 0}\frac{100h+10h^2}{h}$$
$$=\lim_{h\to 0}(100+10h)=100$$

So the instantaneous velocity of the rocket at 5 seconds is 100 ft/s.

REMARK Notice that to take the average velocity, you do not take the limit. To take the instantaneous velocity you need to take the limit as $h\to 0$.

ASSIGNMENT #3

For problems 1-5, find the average velocity of an object represented by the equation, $s(t) = 10t^2$, in the given interval where s is measured in feet and t is measured in seconds.

1) $t = 2$ to $t = 2.1$

2) $t = 2$ to $t = 2 + h$

3) $t = 4$ to $t = 4.5$

4) $t = 4$ to $t = 4.1$

5) $t = 4$ to $t = 4 + h$

For problems 6-7, find the instantaneous velocity of an object represented by the equation, $s(t) = 10t^2$, for the given time.

6) $t = 2$

7) $t = 4$

8) Suppose an astronaut drops a rock from a cliff on the moon. If $d = 5.5t^2$ where d is the number of feet the rock travels in t seconds, at what instant is the rock going 55 miles per hour?

9) Suppose a marble initially at rest rolls down an inclined plane. Let d be the number of inches the marble rolls in t seconds where d is related to t by the equation, $d = 3t^2$. Find the average rate of change of d with respect to t over the interval $t = 2$ to $t = 2.5$.

10) Evaluate $\dfrac{f(2+h) - f(2)}{h}$ to find the average rate of change for the equation $d = 3t^2$ from $t = 2$ to $t = 2 + h$. Find instantaneous velocity at $t = 2$.

11) Calculate the instantaneous velocity at $t = 3$ by evaluating $\lim_{h \to 0} \dfrac{f(3+h) - f(3)}{h}$ for the equation $d = 3t^2$.

12) A ball is thrown vertically upward with an initial velocity of 80 ft/sec. If the ball is released 6 feet above the ground, its height in feet, s, at the end of t seconds is given by the formula

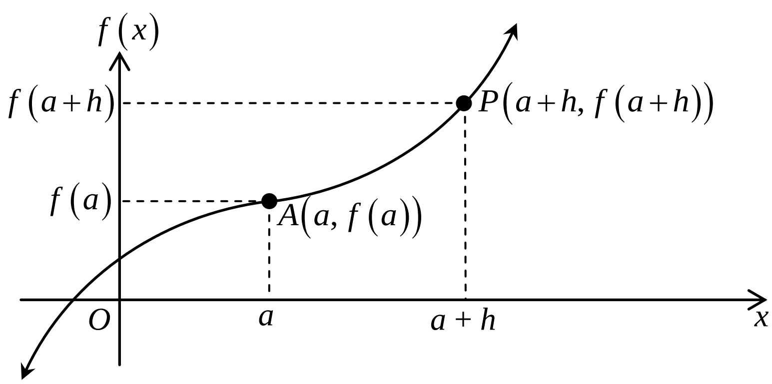
$$s(t) = -16t^2 + 80t + 6.$$

Find the instantaneous velocity of the ball at the end of 0, 1, 2, and 3 seconds. Derive a formula for *v(t)* that fits this data.

Difference Quotient

Difference Quotient	The ratio of the differences in function values to the difference in values of the independent variable is called the difference quotient. $$\frac{f(x+h)-f(x)}{h}$$

The graph below illustrates the difference quotient by showing the changes in the variable and the changes in the value of the function over an interval $a \le x \le a+h$.



Definition: Derivative	The **derivative** is a measure of how a function changes as its input changes. A function is differentiable if the derivative of the function exists at each value of its domain. The derivative is denoted by $f'(x)$. $$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

EXAMPLE 3 Applications of derivative

Blowing up a balloon. If the radius of a spherical balloon is r cm, the volume V cm^3 enclosed by the balloon is given by the equation $V = \frac{4}{3}\pi r^3$. When r changes, the volume V changes.
Find the rate at which V is changing with respect to r when $r = 2$.

SOLUTION

$$\frac{\text{change in volume}}{\text{change in radius}} = \frac{f(2+h) - f(2)}{h}$$

$$\begin{aligned}
\lim_{h\to 0}\frac{f(2+h) - f(2)}{h} &= \lim_{h\to 0}\frac{\frac{4}{3}\pi(2+h)^3 - \frac{4}{3}\pi(2)^3}{h} \\
&= \lim_{h\to 0}\frac{\frac{4}{3}\pi\left(8+12h+6h^2+h^3\right) - \frac{4}{3}\pi(8)}{h} \\
&= \lim_{h\to 0}\frac{\frac{4}{3}\pi\left(12h+6h^2+h^3\right)}{h} \\
&= \lim_{h\to 0}\frac{\frac{4}{3}\pi h\left(12+6h+h^2\right)}{h} \\
&= \lim_{h\to 0} 16\pi + 8\pi h + \frac{4}{3}\pi h^2 \\
&= 16\pi
\end{aligned}$$

Hence, the volume is changing at a rate of $16\pi \frac{\text{cm}^3}{\text{cm}}$ at $r = 2$.

REMARK Notice that $(2+h)^3$ was expanded to $\left(8+12h+6h^2+h^3\right)$ in 1 step by the use of Pascal's Triangle. The first four levels of Pascal's Triangle is given below.

1 ← 0 degree
1st degree → 1 1
1 2 1 ← 2nd degree
3rd degree → 1 3 3 1

EXAMPLE 4 Applications of derivative

A farmer estimates that if he harvests his crop now he will get 50 bushels per acre which he can sell for \$1.00 per bushel. Past experience suggests that his crop will increase at a rate of 5 bushels per week, but the price will probably decline at the rate of 5¢ per week. However, he can wait no longer than 6 weeks or his entire crop may be endangered. When should he harvest his crop so that he gets the maximum amount?

The amount *A* is controlled by the following equation, where *w* represents the number of weeks he should wait before harvesting his crop.

$$A=(50+5w)(1-.05w)=\frac{1}{4}\left(200+10w-w^2\right)$$

The graph of this function is a parabola. The vertex of this parabola will be a maximum, where $f'(w)=0$.

SOLUTION

$$f'(w)=\lim_{h\to 0}\frac{f(w+h)-f(w)}{h}$$

$$=\lim_{h\to 0}\frac{\frac{1}{4}\left[\left(200+10(w+h)-(w+h)^2\right)\right]-\left[\frac{1}{4}\left(200+10w-w^2\right)\right]}{h}$$

$$=\lim_{h\to 0}\frac{\frac{1}{4}\left(10h-2wh-h^2\right)}{h}$$

$$f'(w)=\frac{10-2w}{4}$$

$$0=\frac{10-2w}{4}$$

$$w=5$$

Thus, the farmer should wait 5 weeks to harvest and sell his crop to maximize his profits.

ASSIGNMENT #4

For problems 1-8, find the derivative of each function at the given value of *x*.

1) $f(x) = x^2$ at $x = 1$

2) $f(x) = 2x$ at $x = 5$

3) $f(x) = 2x + 1$ at $x = 4$

4) $f(x) = x^2 + 1$ at $x = 3$

5) $f(x) = 4x^2$ at $x = 2$

6) $f(x) = x^2 + x$ at $x = 7$

7) If $g(x) = x^2 + 4x$, find $g'(2)$

8) If $h(x) = 2x^2 + 1$, find $h'(1)$

Solve each of the following.

9) Find the rate of change of the circumference of a circle with respect to the radius when the radius is 3 cm.

10) Find the rate of change of the area of a square with respect to the length of a side, when the side is 5 cm.

11) Find the rate of change of the volume of a cube with respect to the length of an edge, when the edge is 2 mm.

12) For a marble rolling in a track, the distance *d* cm from one end at time *t* seconds is given by $d = 5t - t^2$. Find the speed of the marble when $t = 2$.

Theorem: Power Rule	To take the derivative of a polynomial function, take the exponent of each term and multiply it by that term's coefficient and the decrease that term's exponent by 1. If $f(x) = x^n$ then $f'(x) = nx^{n-1}$

Definition: Derivative of a constant function	The derivative of a constant function, $f(x) = c$, is zero. $f'(x) = 0$ for all x in the domain of f.

The derivative of a constant c times a function f is the constant times the derivative of the function $c \cdot f'(x)$, for all values of x for which $f'(x)$ exists. Thus if $g(x) = c \cdot f(x)$ then $g'(x) = c \cdot f'(x)$.

EXAMPLE 5 Find the derivative

Find the derivative of $f(x) = x^6$

<u>SOLUTION</u>

$$f'(x) = nx^{n-1}$$
$$= 6x^{6-1} = 6x^5$$

EXAMPLE 6 Find the derivative

Find the derivative of $f(x) = 4x^3 - 2x + 6$

<u>SOLUTION</u>

$$f'(x) = (3)4x^{3-1} - (1)2x^{1-1} + 0$$
$$= 12x^2 - 2$$

ASSIGNMENT #5

For problems 1-5, find the derivative of each function.

1) $f(x) = (x+3)^2$

2) $f(x) = 6 - 4x^5 + 2x^9$

3) $f(x) = (x^2 - 3x)^2$

4) $f(x) = (x+3)(x-3)(2x+5)$

5) $f(x) = 3 + x - x^2$; *Find* $f'(0),\ f'\left(\frac{1}{2}\right),\ f'(1),\ f'(-10)$

For problems 6-8, find the derivative of each function. Sketch the graph of *f* and f'.

6) $f(x) = 5x$

7) $f(x) = x^2$

8) $f(x) = x^3$

For problems 9-11, use the function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 6x$ to find *x* to make each of the following true.

9) $f'(x) = 0$

10) $f'(x) = -4$

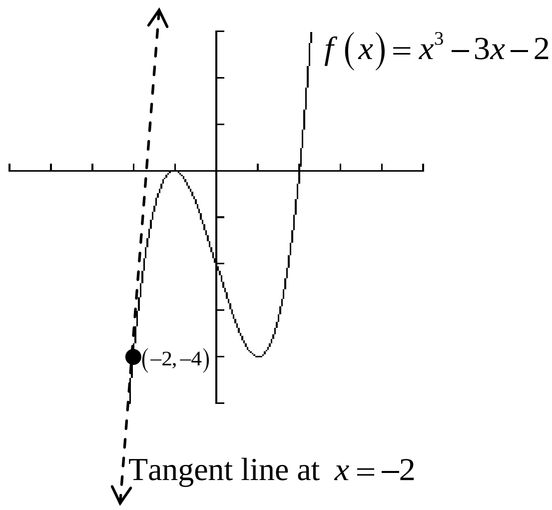
11) $f'(x) < 0$

12) Given the function $f(x) = (x^3 - 2)^2$, find the derived function f' and the rate of change of *f* at $x = -1$ and $x = 2$.

Interpretation of Derivative

By taking *h* sufficiently small, the derivative of a function at a specific *x* can be interpreted as the equation to find the slope of the tangent line to the curve at that specific point.

For the equation $f(x) = x^3 - 3x - 2$ the point $(-2, -4)$ is graphed to the right. To find the slope of the tangent line $f(x) = x^3 - 3x - 2$ at $x = -2$ you need find $f'(-2)$. $f'(x) = 3x^2 - 3$ will find the slope of the tangent line of any point in the domain. So $f'(-2) = 9$, so the slope of the tangent line of $f(x) = x^3 - 3x - 2$ at $x = -2$ is 9.



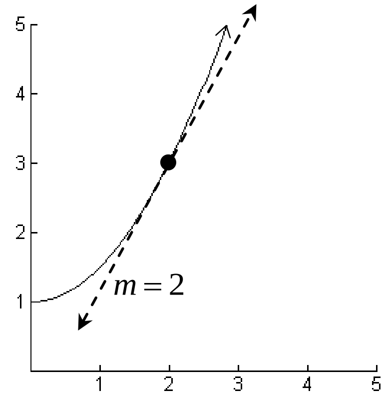
Many applications of calculus deal with a change in some variable. To show this, the symbol *h* is often replace by Δx. It is read *delta x*. Thus Δx is equal to the change, or increment, in *x*. The corresponding change in the value of *y* is denoted by Δy. The delta notation then may be used for the derivative.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

Gottfried Leibniz introduced alternative symbols for the derivative: $f'(x) = \frac{dy}{dx}$. The notation $\frac{dy}{dx}$, called **differential notation**, was one of the original notations used for derivative. It is still widely used. Since *dy* and *dx* can be given individual meaning, differential notation makes certain formulas easy to remember.

EXAMPLE 7 Find slope of the tangent line to a curve at a given point.

Find the slope of the tangent line of $f(x) = \frac{1}{2}x^2 + 1$ at $x = 2$



SOLUTION

$$\frac{dy}{dx} = (2)\frac{1}{2}x^{2-1} + 0$$

$$\frac{dy}{dx} = x$$

So at the point where $x = 2$, the slope is of the tangent line is 2.

ASSIGNMENT #6

For problems 1-6, find the equation of the tangent line of each of the following functions at the given point.

1) $y = 2x^4$ at $x = -1$

2) $y = (2x+3)(x-1)$ at $x = 3$

3) $y = 2x^5$ at $x = 1$

4) $y = 1 - x^2$ at $x = 0$

5) $y = 5$ at $x = 2$

6) $y = x$ at $x = -1$

For problems 7-9, solve each of the following.

7) Show that there is one tangent to the curve $y = x^2 + 5$ which has a slope of 4.
Find its equation.

8) Find the coordinates of the point on the curve $y = x^2 + 4x + 6$ at which the tangent has a slope of 12.

9) Find the equation of the tangent line to the curve $y = x^3$ at the point where $x = 1$. Find the point at which the tangent line meets the curve again.
(HINT: Show the tangent line has equation $y = 3x - 2$ and meets the curve where $x^3 = 3x - 2$.)

ASSIGNMENT #7

For problems 1-9 find the derivative.

For problems 10-17, write each expression as a sum of terms in the form of ax^n. Then find the derivative.

No fraction in fractions, no fractional exponents, no negative exponent in answers.

1) $\frac{1}{x^4}$

2) $\frac{1}{\sqrt{x}}$

3) $\frac{2}{x^3}$

4) $\frac{1}{2x^{\frac{1}{2}}}$

5) $\frac{2}{3x^2}$

6) $\frac{4}{3x^3}$

7) $\frac{1}{5x^4}$

8) $\frac{1}{2\sqrt{x}}$

9) $\frac{2}{x}$

10) $\sqrt{x}+\frac{1}{\sqrt{x}}$

11) $2x^2-\frac{1}{4x^2}$

12) $\frac{x}{5}+\frac{5}{x}$

13) $8x^{\frac{3}{4}}-\frac{6}{x^{\frac{2}{3}}}$

14) $x^2\left(1+\sqrt{x}\right)$

15) $\left(x^2-\frac{1}{x^2}\right)^2$

16) $\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2$

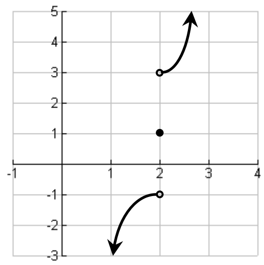
17) $\frac{2x^3-3x^2+4}{x^3}$

18) If $f(x)=4x^{\frac{3}{2}}$, find the values of $f^{'}(0)$, $f^{'}(1)$, $f^{'}(4)$ and $f^{'}\left(\frac{1}{9}\right)$.

Rate of Change Review W.S. **Name**____
No Graphing Calculator

1-10) FIND EACH LIMIT. IF THE LIMIT DOES NOT EXIST, WRITE: **DNE**

1) $\lim_{x\to 2} f(x) =$ ________



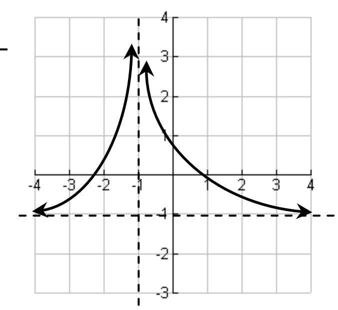
2) $\lim_{x\to -4} \dfrac{x+4}{x^2-16} =$ ________

3) $\lim_{x\to 1} \dfrac{x^2+4x}{3} =$ ________

4) $\lim_{x\to \infty} \sqrt[3]{x} =$ ________

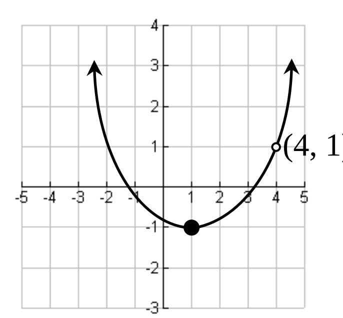
5) $\lim_{x\to \infty} \dfrac{5x^2-2}{7-3x^2} =$ ________

6) $\lim_{x\to -1} f(x) =$ ________



7) $\lim_{n\to \infty} \dfrac{2n+3}{n^2} =$ ________

8) $\lim_{n\to 4} f(x) =$ ________



9-14) FIND THE DERIVATIVE. **USE THE SHORTCUTS.**
SIMPLIFY. NO NEGATIVE OR FRACTIONAL EXPONENTS.

9) $f(x) = (2x+1)(3x-2)$

10) $f(x) = \frac{2}{x} + \frac{5}{x^3} - 4x^2$

11) $f(x) = \frac{5x^4 - 8x^3 + 2x - 3}{x}$

12) $f(x) = 4x\sqrt{x} - 3x^2\sqrt{x}$

13) $f(x) = \frac{5}{2x^{-3}}$

14) $f(x) = \frac{x+3}{\sqrt{x}}$

15) Suppose a marble initially at rest rolls down an inclined plane. Let s be the number of inches the marble rolls in t seconds where s is related to t by the equation: $s(t) = 4t^2$

a) Find the average velocity for s with respect for t when t = 2 and t = 5.

b) Calculate the instantaneous velocity at t = 4.

c) At what instant is the marble rolling at $24.296 \ \frac{\text{in}}{\text{sec}}$?

16) $f(x) = x^2 - 3$. Find $f'(x)$ using the definition of derivative. **(NO SHORT CUT)**

17) Find the equation in slope-intercept form of the tangent line to: $y = 4x^2 - 3x + 2$ at $x = 1$.

18) Find the coordinates of the points on the curve $y = x^2(x-3)$ at which the slope of the tangent line is 9.

Solutions to Packet #1

Assignment #3

1) 41 ft/sec
2) $(40+10h)$ ft/sec
3) 85 ft/sec
4) 81 ft/sec
5) $(80+10h)$ ft/sec
6) $\lim_{h\to 0}(40+10h)=40$
7) $\lim_{h\to 0}(80+10h)=80$
8) $7\frac{1}{3}$ seconds
9) 13.5 in/sec
10) Average Rate : $12+3h$ in/sec
    Instantaneous Rate : 12 in/sec
11) 18 in/sec
12) $t=0$; 80 ft/sec
    $t=1$; 48 ft/sec
    $t=2$; 16 ft/sec
    $t=3$; -16 ft/sec
    $v(t)=-32t+80$

Assignment #4

1) 2
2) 2
3) 2
4) 6
5) 16
6) 15
7) 8
8) 4
9) 2π cm/cm
10) 10 cm^2/cm
11) 12 mm^3/mm
12) 1 cm/sec

Assignment #5

1) $2x+6$

2) $-20x^4+18x^8$

3) $4x^3-18x^2+18x$

4) $6x^2+10x-18$

5) $1-2x$

6) $f'(x)=5$

7) $f'(x)=2x$

8) $f'(x)=3x^2$

9) $x=-3,\ x=2$

10) $x=-2,\ x=1$

11) $-3<x<2$

12) $f'(x)=6x^5-12x^2$

$f'(-1)=-18$

$f'(2)=144$

Assignment #6

1) $y=-8x-6$

2) $y=13x-21$

3) $y=10x-8$

4) $y=1$

5) $y=5$

6) $y=x$

7) $y=4x+1$

8) $(4,38)$

9) $(-2,-8)$

Assignment #7

1) $-\frac{4}{x^5}$

2) $-\frac{1}{2x\sqrt{x}}$

3) $-\frac{6}{x^4}$

4) $-\frac{1}{4x\sqrt{x}}$

5) $-\frac{4}{3x^3}$

6) $-\frac{4}{x^4}$

7) $-\frac{4}{5x^5}$

8) $-\frac{1}{4x\sqrt{x}}$

9) $-\frac{2}{x^2}$

10) $\frac{1}{2\sqrt{x}}-\frac{1}{2x\sqrt{x}}$

11) $4x+\frac{1}{2x^3}$

12) $\frac{1}{5}-\frac{5}{x^2}$

13) $\frac{6}{\sqrt[4]{x}}+\frac{4}{x\sqrt[3]{x^2}}$

14) $2x+\frac{5x\sqrt{x}}{2}$

15) $4x^3-\frac{4}{x^5}$

16) $1-\frac{1}{x^2}$

17) $\frac{3}{x^2}-\frac{12}{x^4}$

18) $f'(0)=0$

$f'(1)=6$

$f'(4)=12$

$f'\left(\frac{1}{9}\right)=2$

Review W.S.

1) DNE

2) $-\frac{1}{8}$

3) $\frac{5}{3}$

4) DNE

5) $-\frac{5}{3}$

6) DNE

7) 0

8) 1

9) $f'=12x-1$

10) $f'=-\frac{2}{x^2}-\frac{15}{x^4}-8x$

11) $f'=15x^2-16x+\frac{3}{x^2}$

12) $f'=6\sqrt{x}-\frac{15x\sqrt{x}}{2}$

13) $f'=\frac{15x^2}{2}$

14) $f'=\frac{1}{2\sqrt{x}}-\frac{3}{2x\sqrt{x}}$

15) $a)$ 28 in / sec $b)$ 32 in / sec $c)$ at 3.037 sec

16) $f'=2x$

17) $y=5x-2$

18) $(3,0),(-1,-4)$