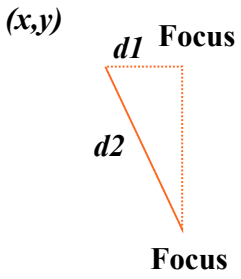
9.3 Hyperbolas

What will you learn?

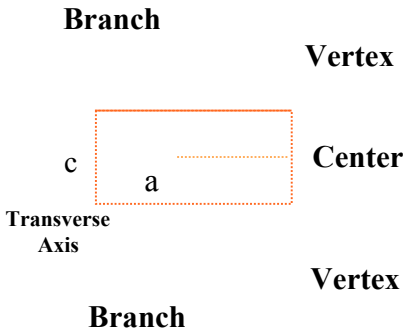
- **Write the equation of hyperbolas in standard form**
- **Find asymptotes of hyperbolas**
- **Use properties of hyperbolas to solve real-life problems**
- **Classify conics frm their general equation**

Definition of a Hyperbola

A hperbola is the set of all points (x, y) in a plane, the difference of whose distances from two distinct fixed points (foci) is a positive constant.



d2-d1 **is a positive constant**



The graph of the hyperbola has 2 disconnected parts - BRANCHES

The line through the 2 foci intersects the hyperbola at 2 points - VERTICES

The line segment connecting the vertices - TRANSVERSE AXIS

The midpoint of the transverse axis - CENTER

Standard Form of a Hyperbola

Center (h, k)

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$ **Transverse Axis is Horizontal**

$$\frac{(y-k)^2}{a^2} - \frac{(h-k)^2}{b^2} = 1$$ **Transverse Axis is Vertical**

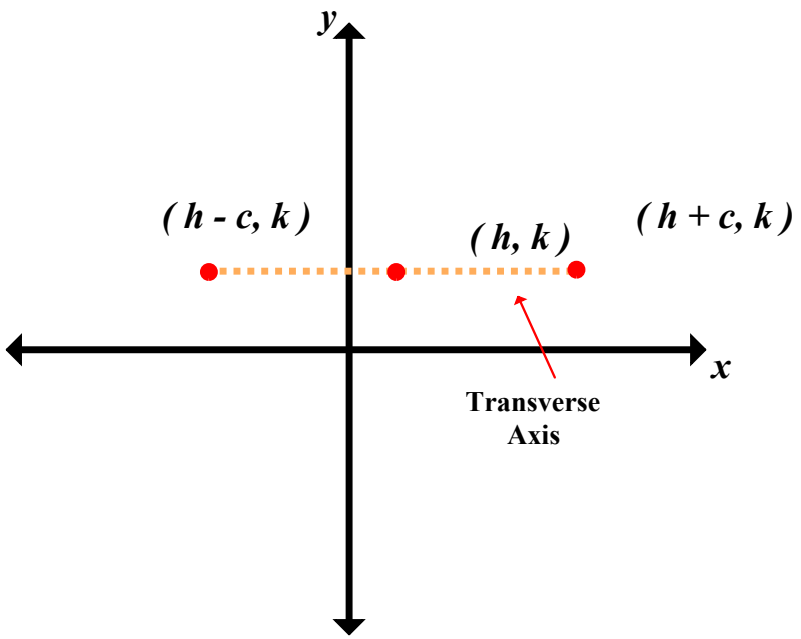
The vertices are *a* units form the center, and the foci are *c* units from the center. Moreover, $c^2 = a^2 + b^2$

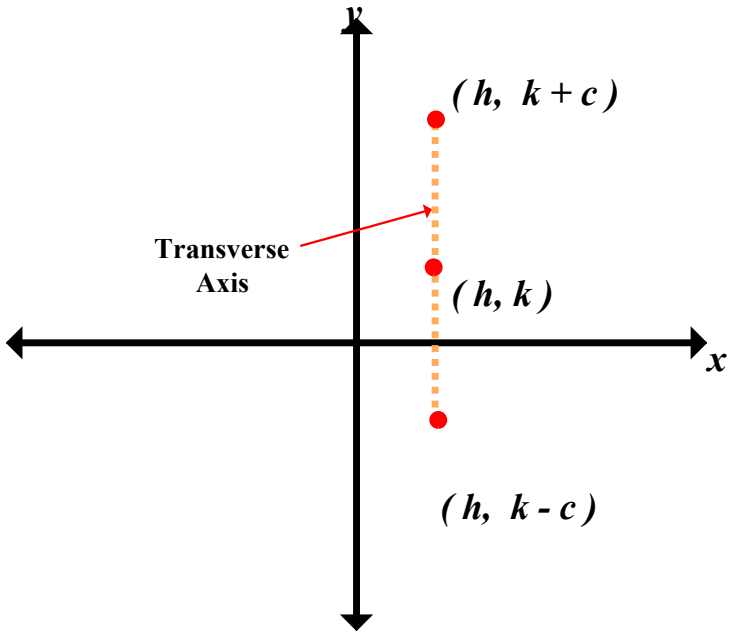
If the center of the hyperbola is at the origin (0, 0) **, the equation takes one of the following forms:**

Center *(0, 0)*

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$ **Transverse Axis is Horizontal**

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$ **Transverse Axis is Vertical**


y
(h - c, k)
(h, k)
(h + c, k)
x
Transverse
Axis


y
(h, k + c)
Transverse
Axis
(h, k)
x
(h, k - c)

Example 1 - Finding the Standard Equation of a Hyperbola

Foci: ***(-1, 2) and (5, 2)***
Vertices : ***(0, 2) and (4, 2)***

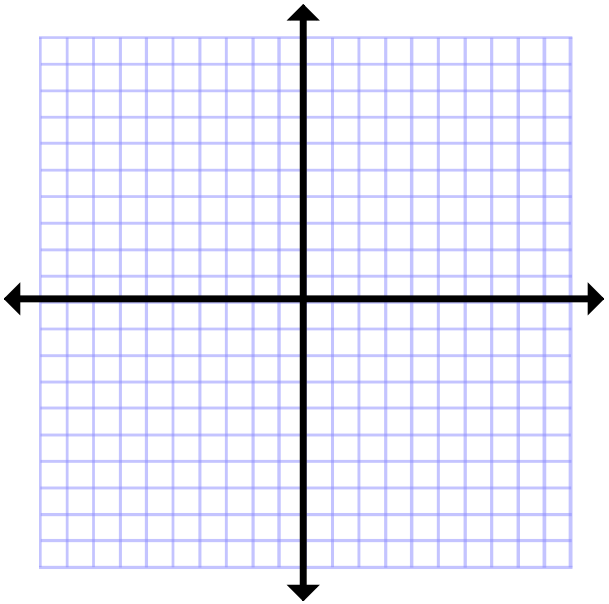
Solution

Find :

Center:_____

c = _____

a + _____



See p. 657; exercise 33

Asymptotes of a Hyperbola

- **Each hyperbola has 2 asymptotes that intersect at the center of the hyperbola.**

- **The asymptotes pass through the corners of a rectangle of dimensions *2a* by *2b* with its center at *(h, k)*.**

$$y = k \pm \frac{b}{a}(x - h)$$ **For Horizontal Transverse Axis**

$$y = k \pm \frac{a}{b}(x - h)$$ **For Vertical Transverse Axis**

Conjugate Axis - is the line segment of length *2b* joining *(h, k + b)* and *(h, k - b)*, if the transverse axis is *horizontal*.
is the line segment of length *2b* joining *(h + b, k)* and *(h - b, k)*, id the transverse is *vertical.*

Example 2 - Sketching a Hyperbola

Sketch the hyperbola whose equation is $4x^2 - y^2 = 16$

Algebraic **Graphical**

Center: _____

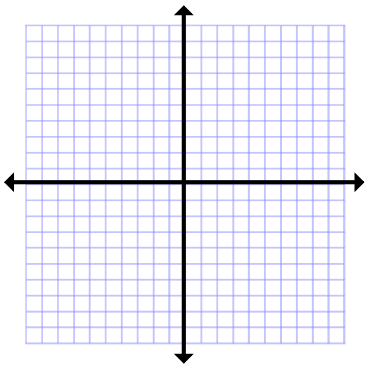
Vertices: _____

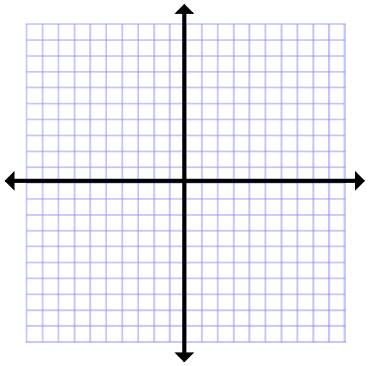
Conjugate Axis End Points: _____

a =_____

b = _____

Asymptotes: _____



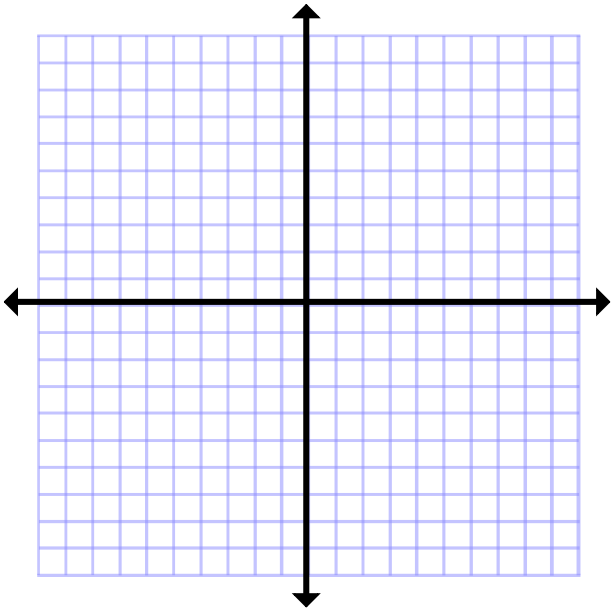


See p. 656; exercise 15

Example 3 - Finding the Asymptotes of a Hyperbola

Sketch the hyperbola given by:

$$4x2 - 3y2 + 8x + 16 = 0$$



Center: _____

Vertices: _____

Conjugate Axis End Points: _____

a =_____

b = _____

Asymptotes: _____

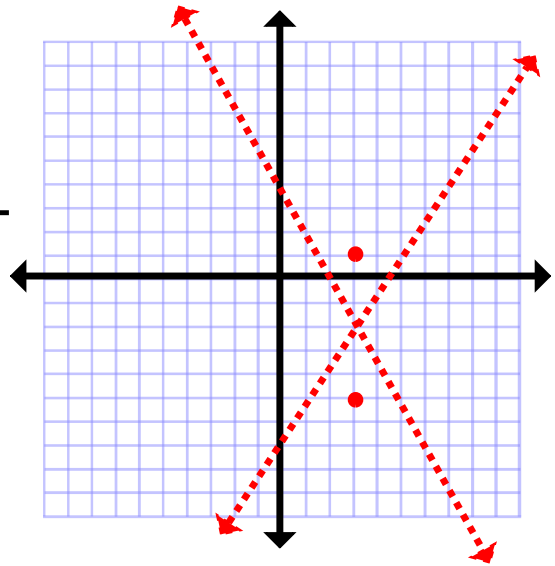
See p. 656; exercise 19

Example 4 - Using Asymptotes to Find the Standard Form

Find the standard form of the equation of the hyperbola having :

Vertices : ___*(3, -5) and (3, 1)*_

Asymptotes : _*y = 2x - 8 and y = -2x + 4*_ _



Center : _____

Transverse Axis : Horizontal Vertical

a **=** _____

b **=** _____

Standard Form:

See p. 657; exercise 39

Eccentricity of a Hyperbola

$$e = \frac{c}{a}$$

If $c > a$: then, $e > 1$

If the eccentricity is LARGE the branches of the hyperbola is nearly FLAT

If the eccentricity is CLOSE TO 1, the branches of the hyperboal are MORE POINTED

Example 5 - An Application Involving Hyperbolas

Two microphones, 1 mile apart, record an explosion.
Microphone A receives the sound 2 seconds before Microphone B.
Where did the expolsion occur?

See p. 657; exercise 43

SEE P. 654 - Orbits of comets

General Equations of Conics

The graph of $Ax2+Cy2+Dx+Ey+F=0$ **is one of the following:**

1. **Circle:** $A = C$, $A \neq 0$
2. **Parabola:** $AC = 0$, $A = 0$ *or* $C = 0$, **but not both!**
3. **Ellipse :** $AC > 0$, A & C **have Like Signs**
4. **Hyperbola:** $AC < 0$, A & C **have Unlike Signs**

The test above is valid if the equation is a conic.
The test does not apply to equations such as $x2+y2=-1$

Example 6 - Classifying Conics from General Equations

a.) $4x2 - 9x + y - 5 = 0$

b.) $4x2 - y2 + 8x - 6y + 4 = 0$

c.) $2x2 + 4y2 - 4x + 12y = 0$

d.) $2x2 + 2y2 - 8x + 12y + 2 = 0$