1.2 Functions

What will you learn?

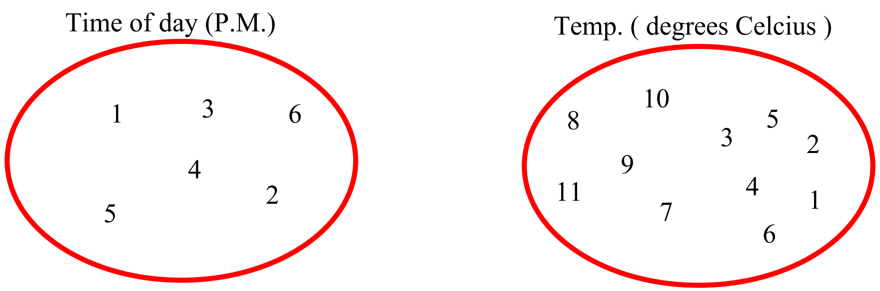
- Whether relations between 2 variables represent a function
- Use function notation and evaluate functions
- Find the domains of functions
- Use functions to model and solve real-life problems
- Evaluate difference quotients

Definition of Function

A **function** f **from a set A to a set B is the relation that assigns to each element** x **in the set A** **exactly one** **element** y **in the set B.**

Domain - (set of inputs) is the set A

Range - (set of outputs) is the set B



Ordered Pairs:

Characteristics of a Function from Set A to Set B

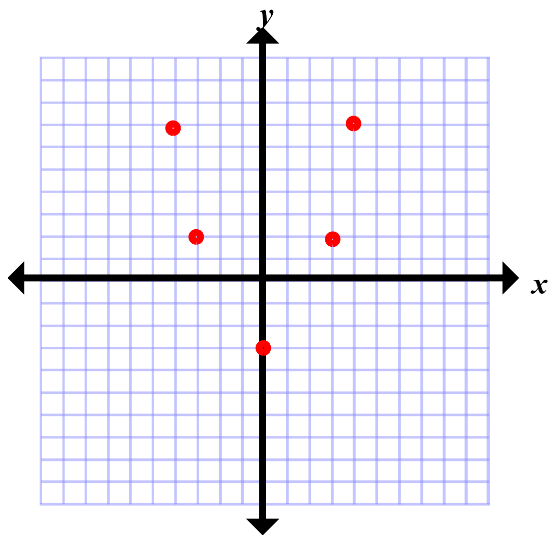
1. **Each element of A must be matched with an element of B**
2. **Some elements of B *may not* be matched with any element of A**
3. **Two or more elements of A may be matcked with the same element of B**
4. **An elements of A (domain) cannot be matched with 2 different elements of B**

Example 1 - Testing for Functions

Decide whether the relation represents *y* as a function of *x*

Table

Input *x*	2	2	3	4	5
Output *y*	11	10	8	5	1



Graph

See p. 24 : exercises 1-8

$$y = x2$$

y is a function of x

Independent Variable :

Dependent Variable :

Example 2 - Testing for Functions represented Algebraically

Which of the equations represent(s) y as a function of x. **(Hint: Solve for y in terms of x)**

a.) $x2 + y = 1$

b.) $-x + y\,2 = 1$

See p. 25 : exercise 19

Function Notation

Input	Output	Equation
x	$f(x)$	$f(x) = 1- x2$

"*f* of *x*"

name — input

Independent Variable:

Dependent Variable:

$$f(x) = 3 - 2x$$

$x = -1 \longrightarrow f(-1) =$

$x = 0 \longrightarrow f(0) =$

Other ways to name functions

$f(t)$ $g(x)$

Example 3 - Evaluating a Function

Let $g(x) = -x2 + 4x + 1$, **find**

a.) $g(2)$

b.) $g(t)$

c.) $g(x + 2)$

Note: $g(x + 2) \neq g(x) + g(2)$

See p. 25; exercise 33

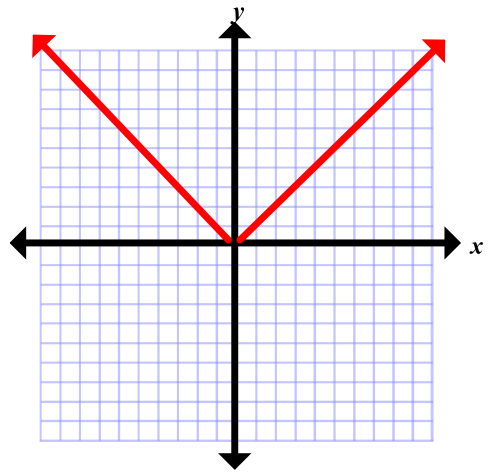
Piecewise-Defined Functions

Piecewise-Defined Functions: a function that is defined by two or more equations over a specified domain.

Example : The Absolute Value Function

$$f(x) = |x|$$

$$f(x) = |x| \begin{cases} x, & x \geq 0 \\ -x, & x \leq 0 \end{cases}$$



Domain :____________

Range:______________

Intercept:____________

Decreasing on :________

Increasing on :________

Evaluate at $x = 0, 1, 3$

$$f(x) = \begin{cases} \dfrac{x}{2} + 1, & x \leq 1 \\ 3x + 2, & x > 1 \end{cases}$$

Example 4 - A Piecewise-Defined Function

Evaluate the function when x = -1 and 0

$$f(x) = \begin{cases} x2 + 1, & x < 0 \\ x - 1, & x \geq 0 \end{cases}$$

See p. 25; exercise 37

The Domain of a Function

The domain can be described explicitly or
it can be *implied* by the expression used to define the function.

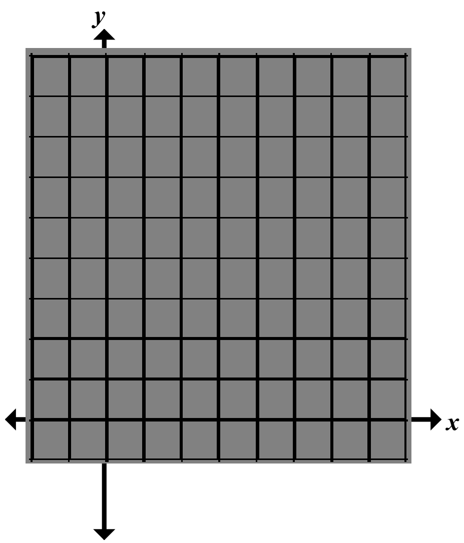
Implied Domain :____

$$f(x) = \frac{1}{x2-\ 4}$$

Implied domain :

$$f(x) = \sqrt{x}$$

Implied domain :



Domain: ___________

Range:_____________

Intercept:___________

Increasing on : _______

See Exploration on P. 20

Example 5 - Finding the Domain of a Function

Find the domain of each function

a.) f : {(-3, 0) (-1, 4) (0, 2) (2 , 2) (4 , -1) }

b.) $g(x) = -3x2 + 4x + 5$

c.) $h(x) = \frac{1}{x+5}$

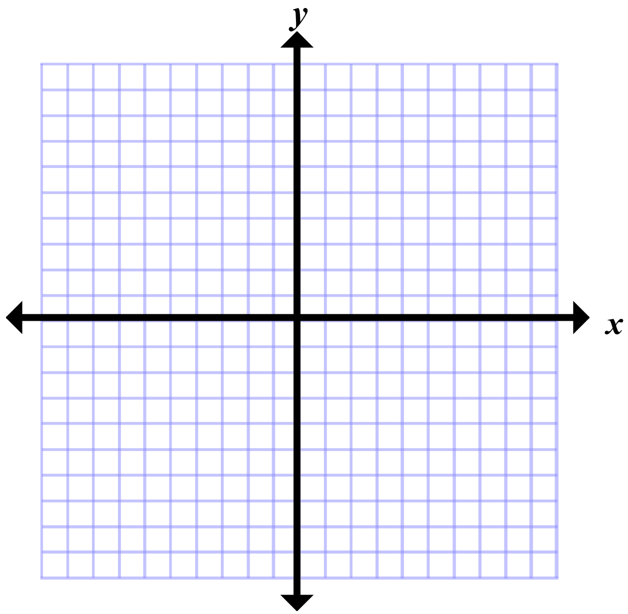
d.) Volume of a Sphere: $V = \frac{4}{3}\pi\, r3$

e.) $k(x) = \sqrt{4 - 3x}$

See p. 26; exercise 51

Example 6 - Finding the Domain and Range of a Function

Use a graphing utility to find the domain and range of the funciton $f(x) = \sqrt{9 - x2}$

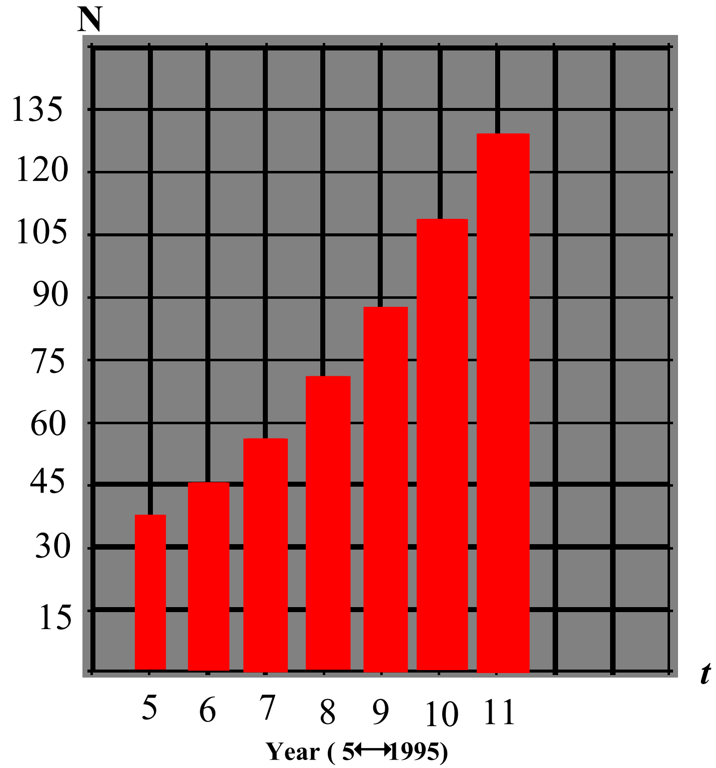


See p. 26 ; exercise 61

Example 7 - Cell Phone Subscribers

The number N (in millions) of cell phone subscribers in the U.S. increased in a linear pattern from 1995 to 1997.
Then, in 1998, the number of subscribers took a jump, and until 2001, increased in a *different linear* pattern.

$$N(t) = \begin{cases} 10.75t - 20.1, & 5 \le t \le 7 \\ 20.11t - 92.8, & 8 \le t \le 11 \end{cases}$$



t = year
$t = 5$ corresponds to 1995

Approximate N for each year form 1995 to 2001

See. p. 28; exercise 79

Example 8 - The Path of a Baseball

A baseball is hit at a point 3 feet above the ground
at a velocity of 100 feet per second and an angle of 45^0.
The path of the baseball is given by the function

$$f(x) = -0.0032x^2 + x + 3$$

where x and y are measured in feet.
Will the baseball clear a 10-foot fence located 300 feet from home plate?

Algebraic Solution

Graphical Solution

See p. 28; exercise 81

Difference Quotients

(basic Calculus definition)

$$\frac{f(x+h)-f(x)}{h}, \qquad h \neq 0$$

Example 9 - Evaluating Difference Quotient

For $f(x) = x2 - 4x + 7$**, find** $\frac{f(x+h)-f(x)}{h}$

See p. 29; exercise 85

Evaluate $f(x) = 2 + 3x - x^2$ for

a.) $f(-3)$

b.) $f(x+1)$

c.) $f(x+h) - f(x)$

Determine if y is a function of x :

$$x^3 + 3x^2y^2 + 1 = 0$$

Find the Domain: $f(x) = \frac{3}{x+1}$