1.5 Combinations of Functions

 **What will you learn?** 

- To add, subtract, multiply & divide functions
- To find compositions of one function with another
- To use combinations of functions to model and solve real-life problems

Arithmetic Combinations of Functions

Two functions can combine to create new functions!

$f(x) = 2x - 3$

$g(x) = x2 - 1$

Let f and g be two functions with overlapping domains.
Then for all x common to both domains :

Example

1. Sum $(f+g)x = f(x) + g(x)$

2. Difference $(f-g) = f(x) - g(x)$

3. Product $(fg)x = f(x) \cdot g(x)$

4. Quotient $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$

Example 1 - Finding the Sum of Two Functions

Given $f(x) = 2x + 1$ **and** $g(x) = x2 + 2x - 1$

Find $(f + g)(x)$.
Then evaluate the sum when $x = 2$.

See p.58; exercise 13

Example 2 - Finding the Difference of Two Functions

Given $f(x) = 2x + 1$ **and** $g(x) = x2 + 2x - 1$

Find $(f - g)(x)$.
Then evaluate the sum when $x = 2$.

Algebraic

Graphical

See p. 58; exercise 15

Remember: domains must overlap
restrictions must be considered

What were the domains of the two functions in Example 1 & Example 2?

What is the domain of the sum /difference of the two functions?

What is the domain of $f(x) = \frac{1}{x}$

What is the domain of $g(x) = \sqrt{x}$

What is the domain of $(f + g)$

Example 3 - Finding the Product of Two Functions

Given $f(x) = x2$ **and** $g(x) = x - 3$

Find $(fg)(x)$.

Evaluate the product at $x = 4$.

See p. 58; exercise 17

Example 4 - Finding the Quotient of Two Functions

Given $f(x) = \sqrt{x}$ **and** $g(x) = \sqrt{4 - x2}$

Find $(f/g)(x)$ **and** $(g/f)(x)$.
Find the domains of f/g **and** g/f.

See p. 53; Graphing Calculator Tech Tip

See p. 58; exercise 19

Compositions of Functions

The composition of the function f with the function g is

$$(f \circ g)(x) = f(g(x))$$

The domain of $f \circ g$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f.

Example 5 - Forming the Composition of f with g

Given $f(x) = \sqrt{x}$, $x \geq 0$

$g(x) = x - 1$, $x \geq 1$

Find $(f \circ g)$ and if possible, find $(f \circ g)(2)$ and $(f \circ g)(0)$

See p. 59; exercise 35

Exploration

$$\text{Let} \quad f(x) = x + 2$$

$$g(x) = 4 - x2$$

Are $f \circ g$ and $g \circ f$ equal?

Example 6 - Compositions of Functions

Let $f(x) = x + 2$

$g(x) = 4 - x2$

Evaluate $(f \circ g)(x)$ and $(g \circ f)(x)$

$x = 0, 1, 2, 3$

Algebraic

Numerical (Graphing Calc)

To determine the domain of $f \circ g$:
restrict the outputs of g so that they are in the domain of f

For example :

Given $f(x) = 1/x$
$g(x) = x + 1$

What are the outputs of g ? _____

What is the domain of f ? _____

What do the outputs of g have to be restricted to? _____

Therefore what is the domain of $f \circ g$? _____

Example 7 - Finding the Domain of the Composite Function

Given $f(x) = x2 - 9$

$g(x) = \sqrt{9 - x2}$

Find $(f \circ g)(x)$

Algebraic

Graphical

See p. 59; exercise 39

Example 8 - A Case in which $f \circ g = g \circ f$

Given $f(x) = 2x + 3$

$g(x) = \frac{1}{2}(x - 3)$

Find $(f \circ g)(x)$ **and** $(g \circ f)(x)$

See p. 59; exercise 43

It is important to be able to identify two functions that make up a composition. "Decompose" a composite function - look for an "inner" and "outer" function.

Example 9 - Identifying a Composition Function

Write the the function $h(x) = (3x - 5)3$ **as a composite of two functions.**

See p. 59; exercise 59

Example 10 - Identifying a Composite Function

Write the function $h(x) = \dfrac{1}{(x-2)2}$ **as a composition of two functions.**

See p. 59; exercise 59

Activities

Given $f(x) = 3x2 + 2$ and $g(x) = 2x$

1. Find $(f+g)(-1)$ and $\left(\frac{f}{g}\right)(2)$

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2. Find $f \circ g$

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3. Find two functions f and g such that $(f \circ g)(x) = h(x)$

See Exploration on p. 56

Example 11 - Bacteria Count

The number N of bacteria in a refrigerated food is given by

$$N(T) = 20T^2 - 80T + 500, \qquad 2 \le T \le 14$$

where T = Temperature of food (Celcius).

When the food is removed from refrigeration, the temperature of the food is given by

$$T(t) = 4t + 2, \qquad 0 \le t \le 3$$

where t = time (hours)

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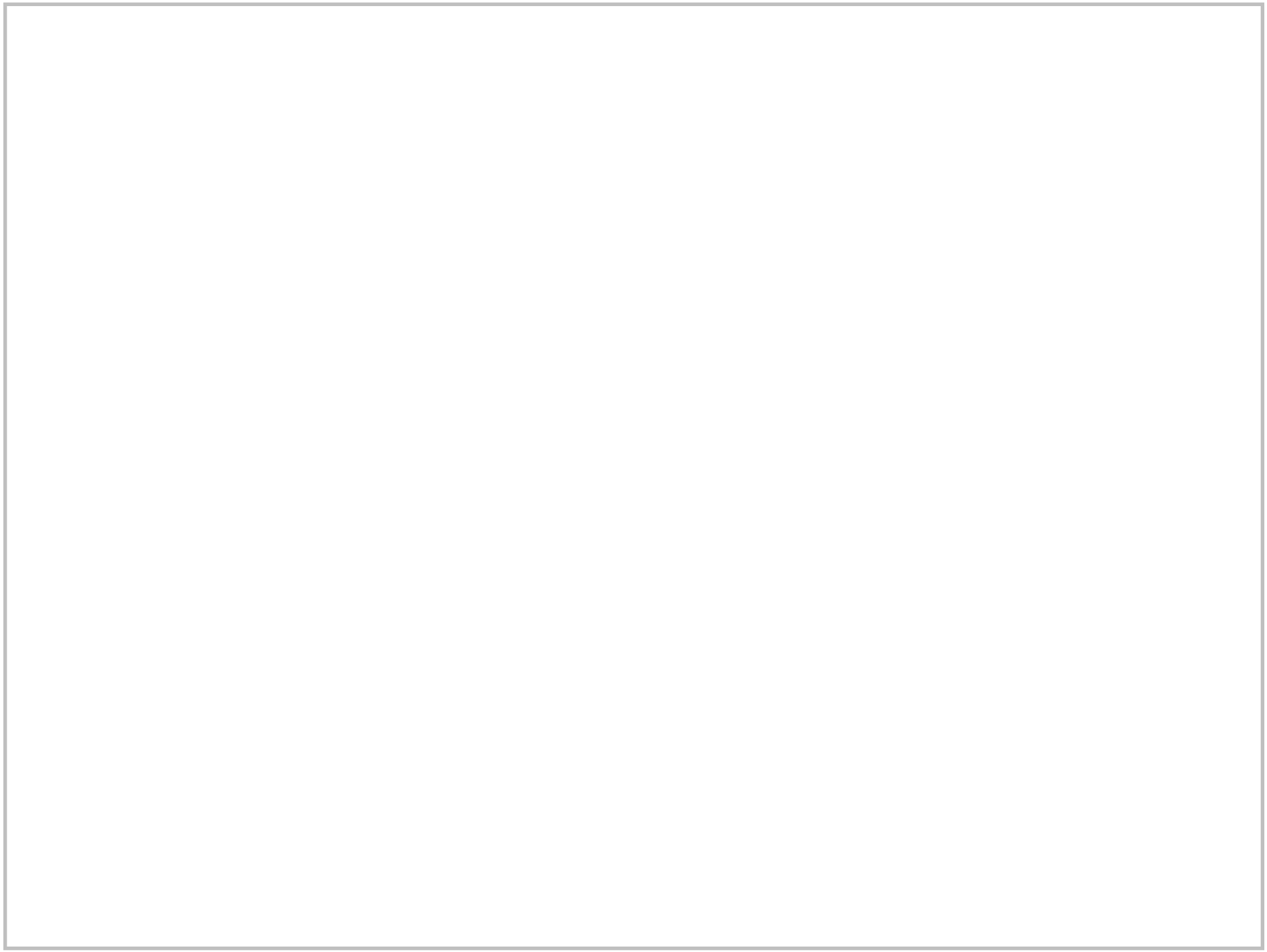
a.) Find the composition of $N(T(t))$

b.) Find the number of bacteria when $t = 2$

c.) Find the time when the bacteria count reaches 2000

See Exploration on p. 57

See p. 60; exercise 79



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