

9.1 Intro to Conics: Parabolas

What will you learn?



- Recognize a conic as an intersection of a plane and a double-napped cone
- Write equations of parabolas in standard form
- Use the reflective property of parabolas to solve real-life problems

Conics

Intersection of a plane and a double-napped cone

Circle

Ellipse

Parabola

Hyperbola

Degenerate Conic: When the intersecting plane passes through the vertex

Point

Line

Two
Intersecting Lines

Parabolas

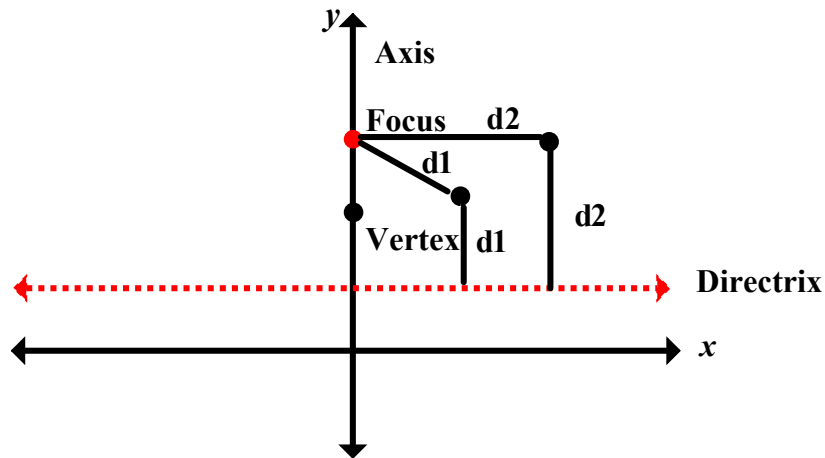
$$f(x) = ax^2 + bx + c$$

Definition of a Parabola

A **Parabola** is the set of all points (x, y) in a plane that are equidistant from a fixed line, the **directrix**, and a fixed point, the **focus**, not on the line.

Vertex - the midpoint between the focus and the directrix
the axis of the parabola

A parabola is symmetric with respect to its axis



Standard Equation of a Parabola

Vertex (h, k)

$$(x - h)^2 = 4p(y - k), \quad p \neq 0$$

Vertical Axis; Directrix: $y = k - p$

$$(y - k)^2 = 4p(x - h), \quad p \neq 0$$

Horizontal Axis; Directrix: $x = h - p$

The focus lies on the axis p units (directed distance) from the vertex.

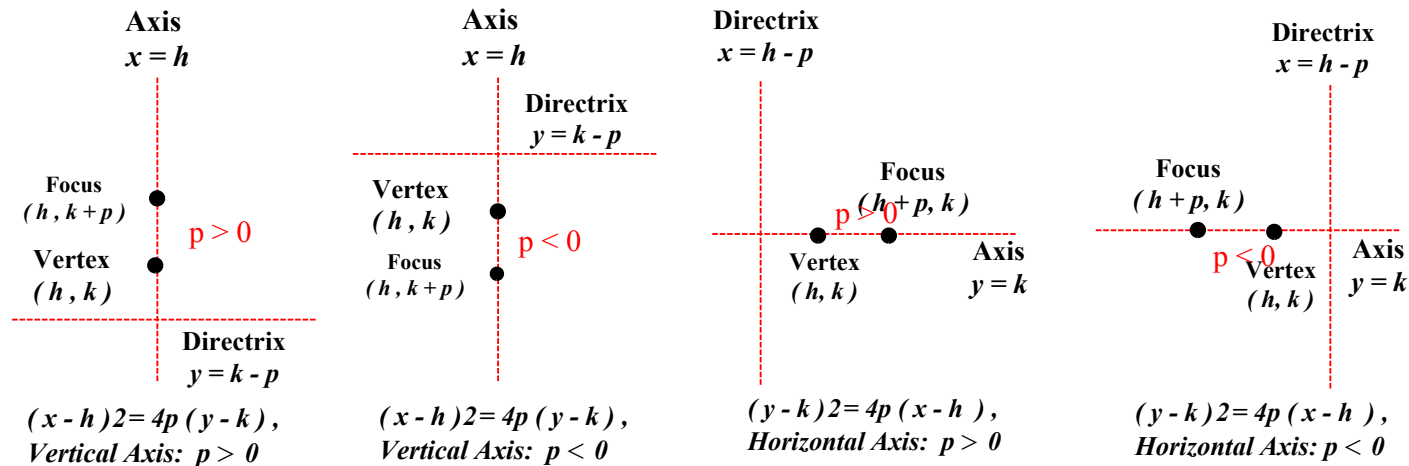
If the *vertex is at the origin*, the equation takes on of the following forms.

$$x^2 = 4py$$

Vertical Axis

$$y^2 = 4px$$

Horizontal Axis



Example 1 - Finding the Standard Equation of a Parabola

Find the standard form of the equation of the parabola with:

Vertex : origin

Focus: $(0, 4)$

See p. 637; exercise 27

Example 2 - Finding the Focus of a Parabola

Find the focus of the parabola given by $y = -\frac{1}{2}x^2 - x + \frac{1}{2}$

See p. 637; exercise 15

Example 3 - Finding the Standard Equation of a Parabola

Find the standard form of the equation of the parabola with:

Vertex (1, 0)

Focus: (2, 0)

See p. 638; exercise 41

Reflective Property of Parabolas

Focal Chord



The line segment that passes through the focus of a parabola and has endpoints on the parabola

Latus Rectum



The specific chord perpendicular to the axis of the parabola

Tangent



A line that touches a single point on the parabola but does not cross the parabola

Parabolic Reflector



Light is reflected in parallel rays

Reflective Property of Parabolas

The tangent line to a parabola at a point P makes equal angles with the following two lines:

1. The line passing through P and the focus
2. The axis of the parabola

Example 4 - Finding the Tangent Line at a Point on a Parabola

Find the equation of the tangent line to the parabola $y = x^2$ at the point $(1, 1)$

