1.6 - Inverse Functions

What will you learn?

- **To find Inverse Functions informally**
- **To verify that two functions are inverses of each other**
- **To use graphs of functions to decide whether they have inverses**
- **To determine if functions are one-to-one**
- **To find Inverse Functions *algebraically***

Do remember how to find the inverse of a relation?

Relation : { (-1, 2) (3, 6) (0, 3) (-4, -1) (4, 7) }

Inverse : _____

What did you do to find the inverse? _____

Inverse Function

"Inverse of f" ⟶ f^{-1}

$$f(x) = x + 4$$

Set A = { 1,2,3,4} **Set B ={5,6,7,8}**

Ordered Pairs: _____

Inverse: _____

Inverse Function: $f^{-1}(x) =$ _____

What do you notice about the function and its inverse function?

_____

Find the composition of the two functions

$f(f^{-1}(x))$

$f^{-1}(f(x))$

What do you notice??

_____

Example 1 - Finding Inverse Functions Informally

Given $f(x) = 4x$

Find $f^{-1}(x)$

Verify that both $f(f^{-1}(x))$ **and** $f^{-1}(f(x))$ **are equal to the identity function**

See p. 69; exercise 1

Example 2 - Finding Inverse Functions Informally

Given $f(x) = x - 6$

Find $f^{-1}(x)$

Verify that both $f(f^{-1}(x))$ **and** $f^{-1}(f(x))$ **are equal to the identity function**

See p. 69; exercise 3

Another way to visualize inverse functions

x	-2	-1	0	1	2
$f(x)$	-8	-7	-6	-5	-4

x	-8	-7	-6	-5	-4
$f^{-1}(x)$	-2	-1	0	1	2

Definition of Inverse Function

Let f and g be two functions such that

$f(g(x)) = x$, **for every x in the domain of g**

and

$g(f(x)) = x$ **for every x in the domain of f**

g is the inverse of f g $f^{-1}(x)$

The domain of f must be equal to the range of f^{-1}

If g is the inverse of f then f must be the inverse of g

Example 3 - Verifying Invers Functions Algebraically

Show that the following functions are inverses of each other.

$$f(x) = 2x3 - 1 \qquad g(x) = \sqrt[3]{\frac{x+1}{2}}$$

See Tech Tip p.64

See p. 69; exercise 15

Example 4 - Verifying Inverse Functions Algebraically

Which of the following functions is the inverse of $f(x) = \frac{5}{x - 2}$

$$g(x) = \frac{x - 2}{x} \qquad h(x) = \frac{5}{x} + 2$$

See p. 69; exercise 19

The Graph of an Inverse Functions

The graphs of a function *f* and its inverse *f -1*are related to each other in the following way:

If point *(a, b)* lies on *f*

Then point *(b, a)* must lie on *f -1*

and vice versa

This means that the graph of *f -1* is a RELECTION of *f* over

$$y = x$$

Example 5 - Verifying Inverse Functions Graphically and Numerically

$$f(x) = 2x^3 - 1 \qquad g(x) = \sqrt[3]{\frac{x+1}{2}}$$

Verify that *f* and *g* are inverses of each other graphically and numerically. (Use you calculator)

See p. 70; exercise 25

The Existence of an Inverse

To have an inverse, a function must be one-to-one

No two elements in the domain of f correspond to the same element in the range of f

Definition of a One-to-One Function

A function is one-to-one if, for a and b in its domain, $f(a) = f(b)$ implies $a = b$

Existence of an Inverse Function

A function f has an inverse function f^{-1} only if it is one-to one

Is $f(x) = x2$ one -to-one ?

Does $f(x) = x2$ have an inverse function?

Tests for On-to-One Functions

- **Use the HORIZONTAL LINE TEST**
- **If f is *increasing* on its entire domain**
- **If f is *decreasing* on its entire domain**

Example 6 - Testing for One-to-One Functions

Is $f(x) = \sqrt{x} + 1$ one-to-one?

Algebraic **Graphical**

See p. 70; exercise 33

Finding Inverse Functions Algebraically

1. **Horizontal Line Test**

2. **Replace $f(x)$ by y**

3. **Interchange the roles of x and y**

4. **Replace y by f^{-1}**

5. **Verify that f and f^{-1} are inverse functions**
   - **show that domain of f equals range of f^{-1}**
   - **show that range of f is equal to the domain of f^{-1}**
   - **show $f(f^{-1}(x)) = x$**
   - **show $f^{-1}(f(x)) = x$**

See Tech Tip on p. 67

Example 7 - Finding an Inverse Function Algebraically

Find the inverse of $f(x) = \frac{5 - 3x}{2}$

See. p 70; exercise 53

Example 8 - Finding an Inverse Algebraically

Find the inverse of $f(x) = x2 - 4$

Use a graphing utility to graph f **and** f^{-1} **in the same window**

See p. 70; exercise 55

Example 9 - Finding an Inverse Function Algebraically

Find the inverse of $f(x) = \sqrt{2x - 3}$

Use a graphing utility to graph f **and** f^{-1} **in the same window**

See p. 70; exercise 59

Activity

1. **Given** $f(x) = 5x - 7$
   Find $f^{-1}(x)$

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2. **Show that *f* and *g* are inverse functions by showing**
   $f(g(x)) = x$ **and** $g(f(x)) = x$

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3. **Describe the graphs of functions that have inverse functions and show how the graph of a function and its inverse function are related.**