2.2 Polynomial Functions of Higher Degree

What will you learn?

- To *use transformations* to sketch graphs of polynomial functions
- To us the Leading Coefficient Test to determine the behavior of graphs of polynomial funciotns
- To find and use zeros of polynomial functions as sketching aids
- To use Intermediate Value Theorem to help locat zeros of polynomial functions

Graphs of Polynomial Functions

Features

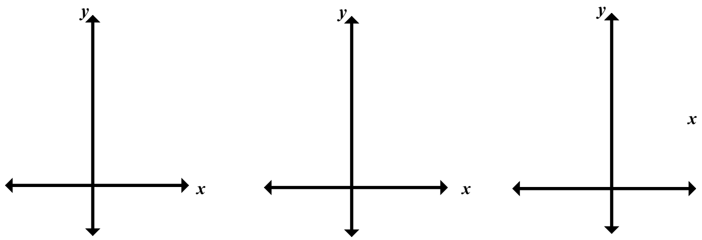
Continuous - ____________________________________

Smooth Turns - __________________________________

Exploration

Use a graphing calc. to graph $y = xn,$ $n = 2,4,8$

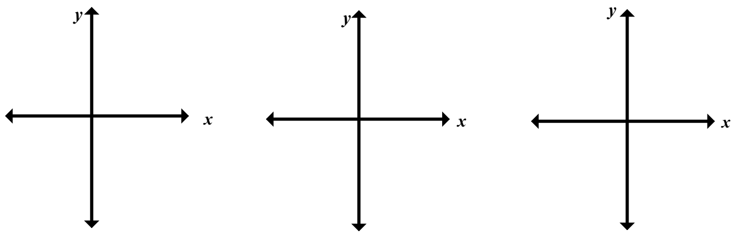
Window: $-1.5 \le x \le 1.5$
$-1 \le y \le 6$



What do you notice???

Use a graphing calc. to graph $y = xn,$ $n = 3,5,7$

Window: $-1.5 \le x \le 1.5$
$-4 \le y \le 4$



What do you notice???

Polynomial Functions:

Degree = 1 → ______________

Degree = 2 → ______________

Degree = n → $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$

n → positive integer

$a_n \neq 0$

Simplest graphs are monomials in the form of $f(x) = x^n$:

If n is *even*, graph is similar to $f(x) = x^2$

If n is *odd*, graph is similar to $f(x) = x^3$

The greater the value of n, the flatter the graph near the origin

Cubic Function

$$f(x) = x3$$

SPolynomials.swf

Domain:________

Range: ________

Intercept:________

Increasing on :_________

Decreasing on: _________

Function: odd even

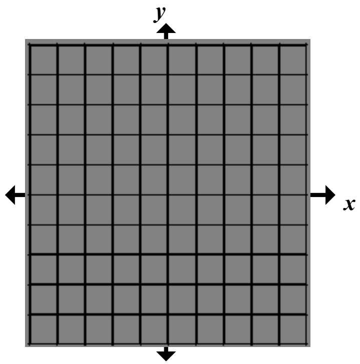
Symmetry: ___________

Example 1 - Transformations of Monomial Functions

Sketch the graph of each function.

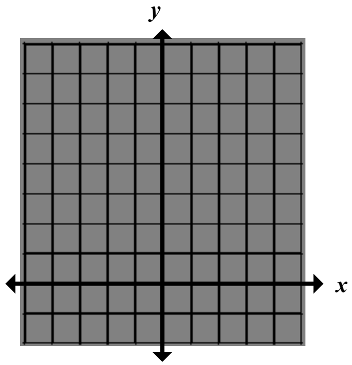
a.) $f(x) = x5$

What do you KNOW about the graph???



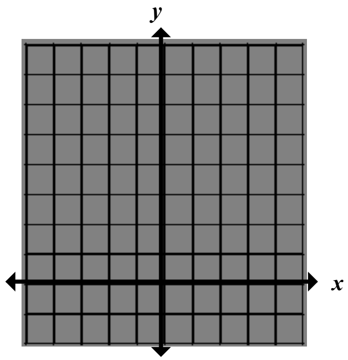
b.) $g(x) = x4 + 1$

What do you KNOW about the graph???
What is the "+ 1" doing to the graph???



c.) $h(x) = (x + 1)4$

What do you KNOW about the graph???
What is the "+1" doing to the graph???



See p. 108; exercise 9

Determining whether a graph eventually rises or falls can be determined by:

- function's degree (even or odd)
- leading coefficient

The Leading Coefficient Test

As x moves without bound to the left or right,
the graph of the polynomial function $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$, $a_n \neq 0$
eventually rises of falls in the following manner:

1. $n \longrightarrow odd$

	Left	**Right**
Leading Coefficient is POSITIVE ($a_n > 0$)		
Leading Coefficient is NEGATIVE ($a_n < 0$)		

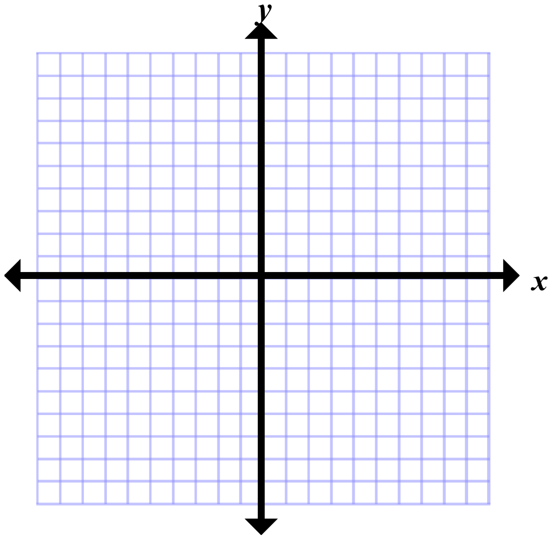
2. $n \longrightarrow even$

	Left	**Right**
Leading Coefficient is POSITIVE ($a_n > 0$)		
Leading Coefficient is NEGATIVE ($a_n < 0$)		

See Exploration on p.101

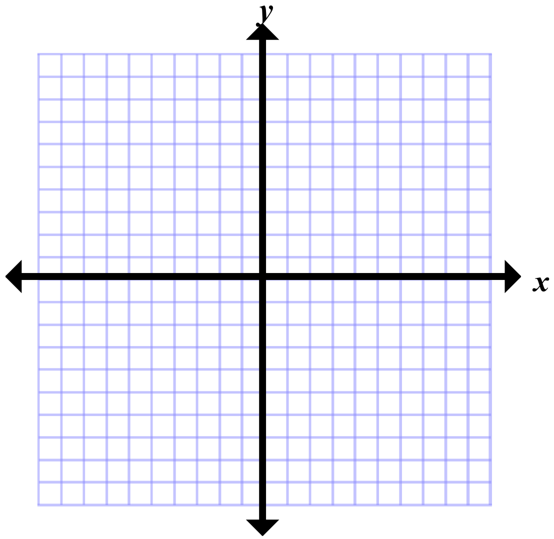
Looking at the following graphs, what can you tell me about

- **the degree (*n*)**
- **leading coefficient (*a*)**



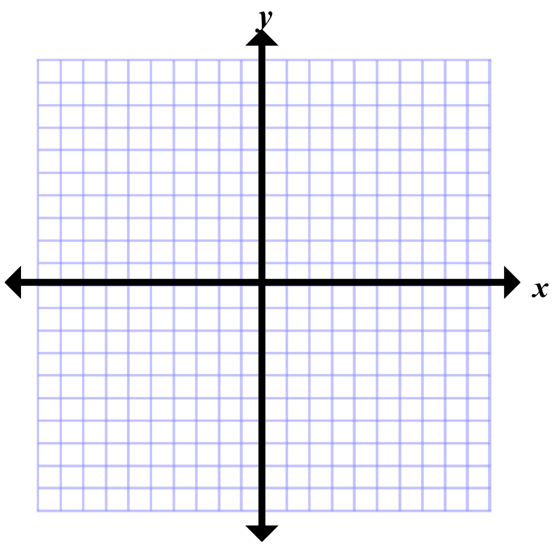
$n =$

$a =$



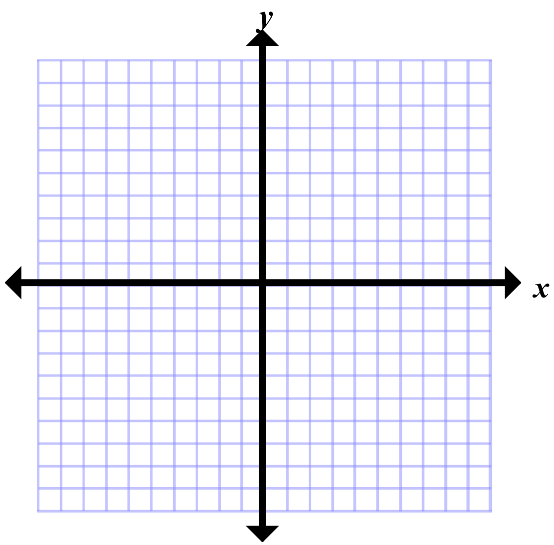
$n =$

$a =$



$n =$

$a =$



$n =$

$a =$

Example 2 - Applying the Leading Coefficient Test

Use the Leading Coefficient Test to describe the *right-hand and left hand behavior* of the graphs of the following functions.

a.) $f(x) = -x^3 + 4x$

b.) $f(x) = x^4 - 5x^2 + 4$

c.) $f(x) = x^5 - x$

See p. 109; exercise 17

Exploration

Using you graphing calculator - count the number of zeros
count the number of relative max & min

What do you notcie????

Use the Leading Coefficient Test to describe the *right-hand and left hand behavior* of the graphs of the following functions.

a.) $f(x) = -x^4 + 2x^2 - 3x$

b.) $f(x) = -x^5 + 3x^4 - x$

c.) $f(x) = 2x^3 - 3x^2 + 5$

Zeros of Polynomial Functions

Given a polynomial function f of degree *n* :

1. **The function *f* has at most *n* real zeros**
2. **The graph of *f* has at most *n -1* relative extrema (max or min)**

Real Zeros of Polynomial Functions

Given *f* is a polynomial function and *a* is a real number:

1. $x = a$ **is a *zero* of *f***
2. $x = a$ **is a solutions of the equation** $f(x) = 0$
3. $(x - a)$ **is a factor of** $f(x)$
4. $(a, 0)$ **is and x- intercept of *f***

Example 3 - Finding the Zeros of a Polynomial Function

Find all zeros of

$$f(x) = x3 - x2 - 2x$$

Algebraic

Graphical

See p. 109; exercise 35

Example 4 - Analyzing a Polynomial Function

Find all real zeros and relative extrema of

$$f(x) = -2x4 + 2x2$$

See p. 109; exercise 47

Repeated Zeros

For a polynomial function, a factor of $(x - a)^k$, $k > 1$, yields a repeated zero $x = a$ of multiplicty k.

1. If k is *odd*, the crosses the x-axis at $x = a$

2. If k is *even*, the graph touches the x-axis at $x = a$

Example 5 - Finding the Zeros of a Polynomial Function

Find all real zeros of

$$f(x) = x^5 - 3x^3 - x^2 - 4x - 1$$

See p. 109; exercise 49

Example 6 - Finding a Polynomial Function with Given Zeros

Find the polynomial functions with the following zeros.

Note: there are many correct answers...WHY?

a.) $-\frac{1}{2}, 3, 3$

b.) $3, 2 + \sqrt{11}, 2 - \sqrt{11}$

See p. 109; exercise 57

Example 7 - Sketching the Graph of a Polynomial Function

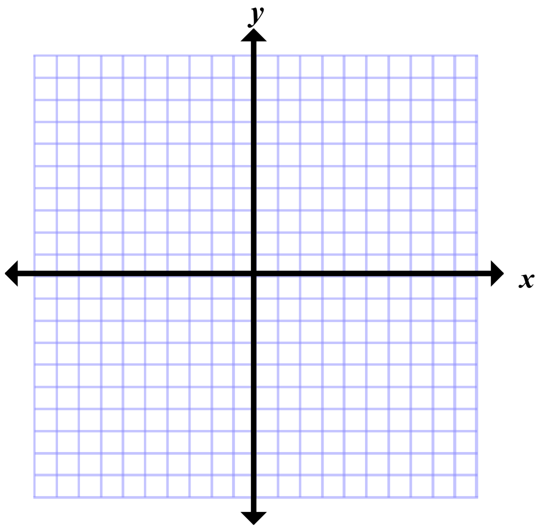
Sketch the graph of

$$f(x) = 3x^4 - 4x^3$$

by hand.

Use everything that you KNOW!

1. **Apply the Leading Coefficient Test**



2. **Find the Zeros of the Polynomial**

3. **Find and Plot a few more points**

4. **Draw the Graph**

See p. 109; exercise 65

Activity

1. **Find all the real zeros of** $f(x) = 6x4 - 33x3 - 18x2$

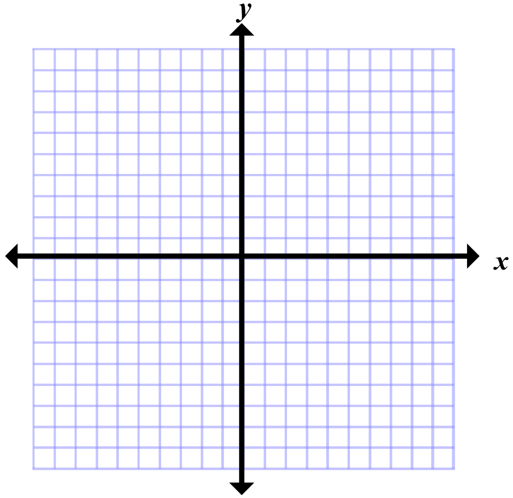
2. **Determine the right & left hand behavior of the function above**

3. **Find a polynomial function of degree 3 that has zeros of 0,2,** $-\frac{1}{3}$

Example 8 - Sketching the Graph of a Polynomial Function

Sketch the graph of

$$f(x) = -2x3 + 6x2 - \frac{9}{2}x$$



1. **<u>Apply the Leading Coefficient Test</u>**

2. **<u>Find the Zeros of the Polynomial</u>**

3. **<u>Find and Plot a few more points</u>**

What do any sign changes in the value of $f(x)$ mean to you??

4. **<u>Draw the graph</u>**

See p. 109; exercise 67

The Intermediate Value Theorem

> **Let *a* and *b* be real numbers such that $a < b$.**
> **If *f* is a polynomial function s.t. $f(a) \neq f(b)$,**
> **then in the interval $[a, b]$, *f* takes on every value**
> **between $f(a)$ and $f(b)$.**

(concerned with the existence of real zeros)

If a value *a* is positive and a value *b* is negative,
then there is at least one real zero *between a and b.*

Example 9 - Approximating the Zeros of a Function

Find three intervals of length 1 in which the poynomial

$$f(x) = 12x3 - 32x2 + 3x + 5$$

is guaranteed to have a zero.

Graphical | **Numerical**