2.6 Rational Functions & Asymptotes

What will you learn?

- To find the domains of rational functions
- To find the vertical & horizontal asymptotes of graphs rational functions
- To use rational functions to model and use real-life problems

Introduction to Rational Functions

Rational Function $\longrightarrow$ $f(x) = \dfrac{N(x)}{D(x)}$

$N(x)$ and $D(x)$ are polynomials and $D(x)$ is not the zero polynomial

Domain: all real numbers except the x-values that make the denominator equal to zero

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Example 1 - Finding the Domain of a Rational Function

Find the domain of $f(x) = 1/x$
and discuss the behavior of f near any excluded x-values

x						
$f(x)$						

x						
$f(x)$						

Basic Reciprocal Function

$$f(x) = \frac{1}{x}$$

Domain:_____

Range:_____

Intercepts:_____

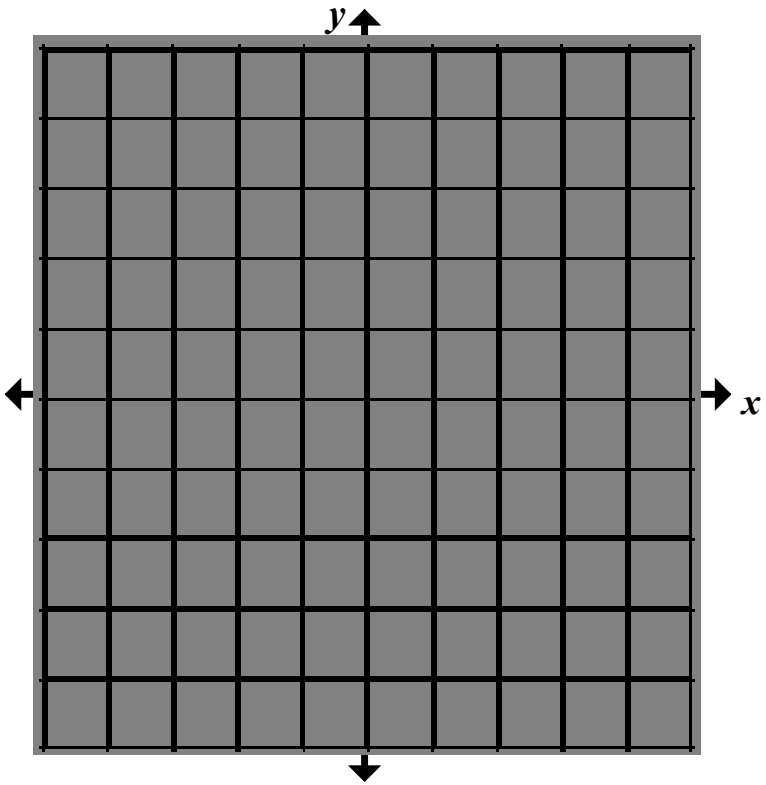
Decreasing on:_____

Function: odd even

Symmetry: _____

Vertical Asymptote:_____

Horizontal Asymptote:_____

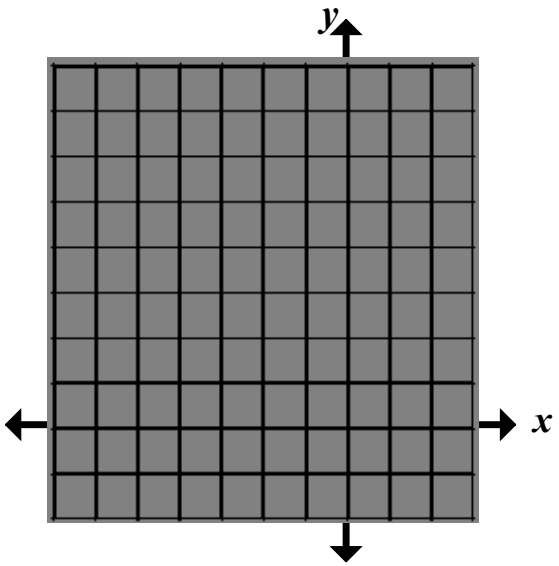


Horizontal & Vertical Asymptotes

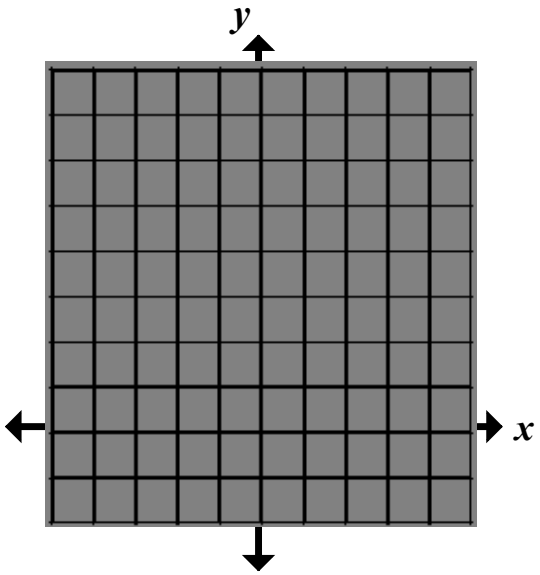
Vertical Asymptote _____

Horizontal Asymptote _____

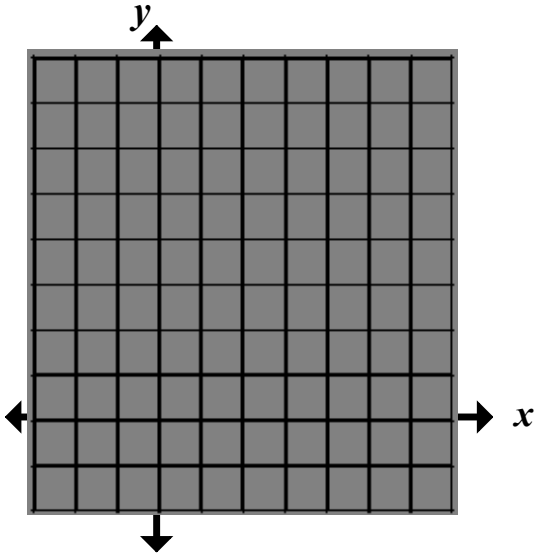
$$f(x) = \frac{2x+1}{x+1}$$



$$f(x) = \frac{4}{x2+1}$$



$$f(x) = \frac{2}{(x-1)2}$$



Asymptotes of a Rational Function

Let f be the rational function

$$f(x) = \frac{N(x)}{D(x)} = \frac{a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \ldots + b_1 x + b_0}$$

where $N(x)$ and $D(x)$ have no common factors

1. The graph of f has vertical asymptotes at the zeros of $D(x)$

2. The graph of f has at most one horizontal asymptote determined by comparing the degrees of $N(x)$ and $D(x)$

   A. If $n < m$, the graph of f has the line y = 0 (x - axis) as the horizontal asymptote

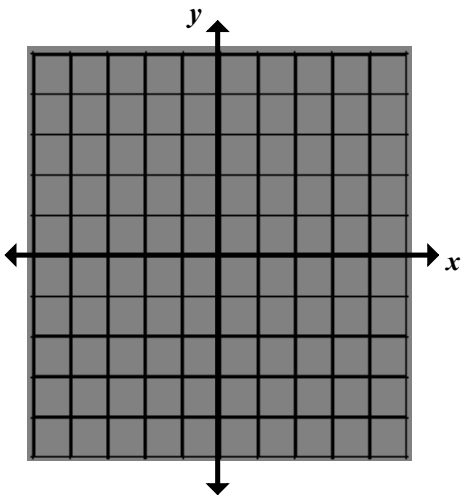
   B. If $n = m$, the graph of f has the line $y = a_n / b_n$ as a horizontal asymptote, where a_n is the leading coefficient of the numerator and b_m is the leading coefficient of the denominator

   C. If $n > m$, the graph has no horizontal asymptote

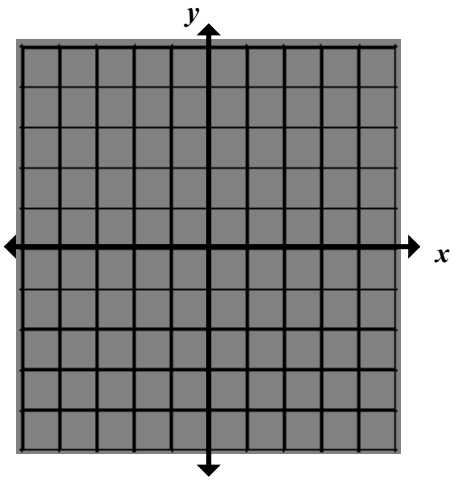
Example 2 - Finding the Horizontal & Vertical Asymptotes

Find all the horizontal & vertical asymptotes of the graph of each rational function.

a.) $f(x) = \frac{2x}{3x2+1}$



b.) $f(x) = \frac{2x2}{x2-1}$

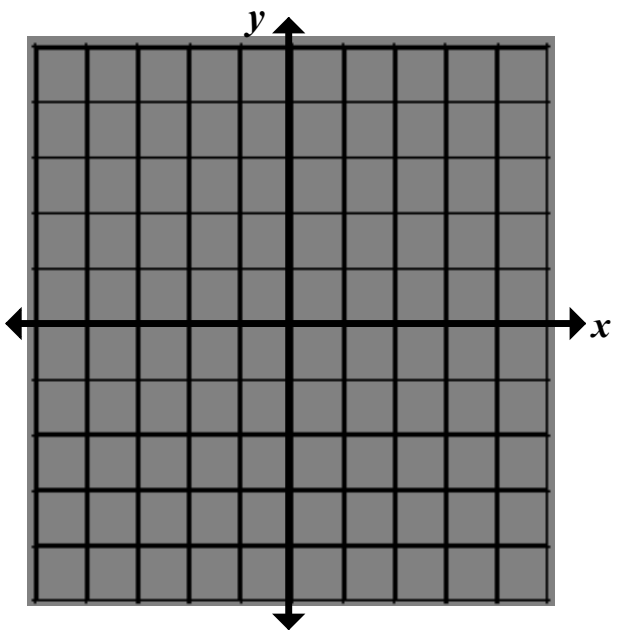


See p. 149; exercise 13

Example 3 - Finding Horizontal & Vertical Asymptotes

Find all horizontal & vertical asymptotes of :

$$f(x) = \frac{x2 - x - 2}{x2 - x - 6}$$



See p. 149; exercise 17

Example 4 - Finding a Function's Domain & Range

Given $f(x) = \dfrac{3x3 + 7x2 + 2}{-4x3 + 5}$

Find:

a.) domain
b.) vertical asymptote
c.) horizontal asymptote

Algebraic **Numerical**

See p. 149; exercise 19

Example 5 - A Graph with 2 Horizontal Asymptotes

A function that is not rational can 2 horizontal asymptotes - one left, one right

$$f(x) = \frac{x + 10}{|x| + 2}$$

$y = -1$ is a horizontal asymptote to the *left*
$y = 1$ is a horizontal asymptote to the *right*

$$f(x) = \begin{cases} \end{cases}$$

See p. 149; exercise 21

Example 6 - Cost-Benefit Model

A utility company burns coal to generate electricity.
The cost C ($) of removing *p%* of the smokestack pollutants is given by

$$C = 80{,}000\,p\,/\,(100-p) \qquad 0 \leq p < 100$$

Graph this function on your calculator.

You are a member of a state legislature that is considering a law that would require utility companies to remove 90% of the pollutants from their smokestack emission.
The current law requires 85% removal.
How much additional cost would there be to the utility company because of the new law?

See p. 149; exercise 35

Example 7 - Ultraviolet Radiation

For a person with sensitive skin, the amount of time T (hours) the person can be exposed to the sun with minimal burning can be modeled by

$$T = \frac{0.37s + 23.8}{s} \qquad 0 < s \leq 120$$

where s is the Sunsor Scale. The Sunsor Scale is based on UVB rays.

See p. 150; exercise 39