2.7 Graphs of Rational Functions

 **What will you learn?** 

- **To *analyze & sketch* graphs of rational functions**
- **To sketch graphs of rational functions that have *slant asymptotes***
- **To use rational functions to model & solve *real-life problems***

Rules for Graphing Rational Functions

Let $f(x) = N(x) / D(x)$, where $N(x)$ and $D(x)$ are polynomials

1. Simplify f

2. Find & plot y-intercepts $\rightarrow f(0)$

3. Find the zeros $\longrightarrow N(x) = 0$

4. Find and sketch any vertical asymptotes $\longrightarrow D(x) = 0$

5. Find & sketch any horizontal asymptotes

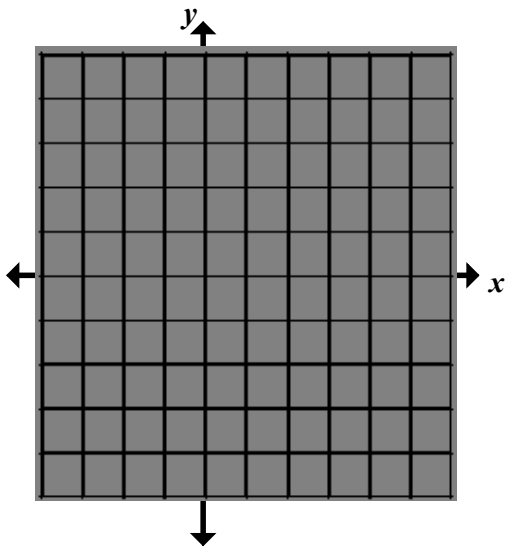
6. Plot at least one point *between* & one point *beyond*
   each x - intercepts and vertical asymptote

7. Use smooth curnves to complete the graph

Tech Tip - use Dot Mode when vertical asymptotes are present

Example 1 - Sketching the Graph of a Rational Function

Sketch (by hand) the graph of $g(x) = \dfrac{3}{x - 2}$



***y* - intercept : _____**

***x* - intercept : _____**

Vertical Asymptote : _____

Horizontal Asymptote : _____

Additional Points :

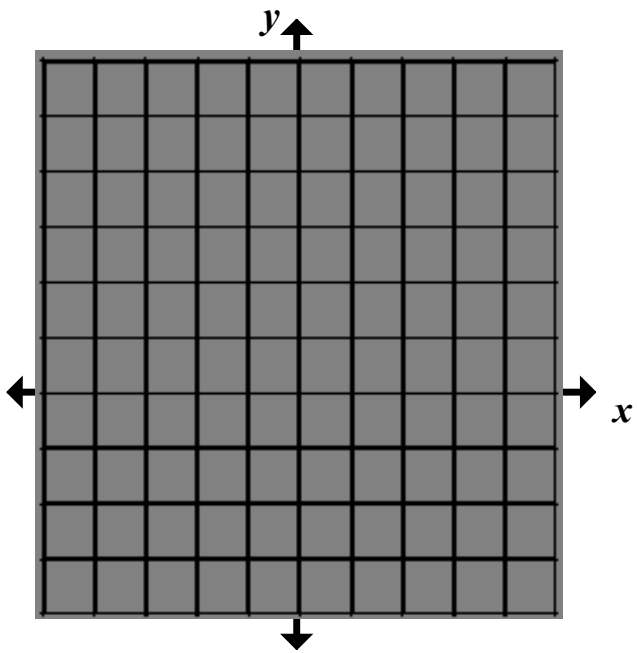
x	$g(x)$

See p. 157; exercise 9

What do you notice about the graph of the above function?

Example 2 - Sketching the Graph of a Rational Function

Sketch (by hand) the graph of $f(x) = \dfrac{2x - 1}{x}$



***y* - intercept : _____**

***x* - intercept : _____**

Vertical Asymptote : _____

Horizontal Asymptote : _____

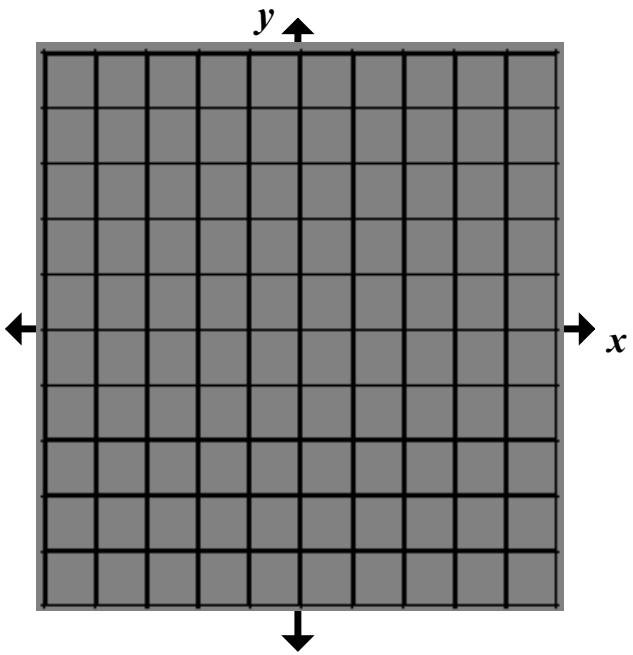
Additional Points :

x	$f(x)$

See p. 157; exercise 13

Example 3 - Sketching the Graph of a Rational Function

Sketch (by hand) the graph of $f(x) = \dfrac{x}{x2 - x - 2}$



y **- intercept : _____**

x **- intercept : _____**

Vertical Asymptote : _____

Horizontal Asymptote : _____

Additional Points :

x	$f(x)$

See p. 157; exercise 21

Example 4 - Sketching the Graph of a Rational Function

Sketch the graph of $f(x) = \dfrac{x2 - 9}{x2 - 2x - 3}$

y - intercept : _____

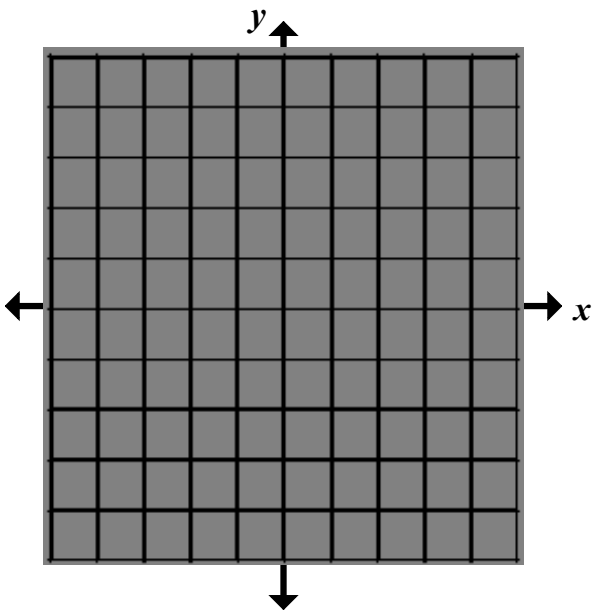
x - intercept : _____

Vertical Asymptote : _____

Horizontal Asymptote : _____

Additional Points :

x	*f* (*x*)



See p. 157; exercise 23

Slant Asymptotes

Slant (Oblique) Asymptote - _____

Example : $f(x) = \frac{x2 - x}{x + 1}$

Example 5 - A Rational Function with a Slant Asymptote

Sketch the graph of $f(x) = \dfrac{x2 - x - 2}{x - 1}$

***y* - intercept : _____**

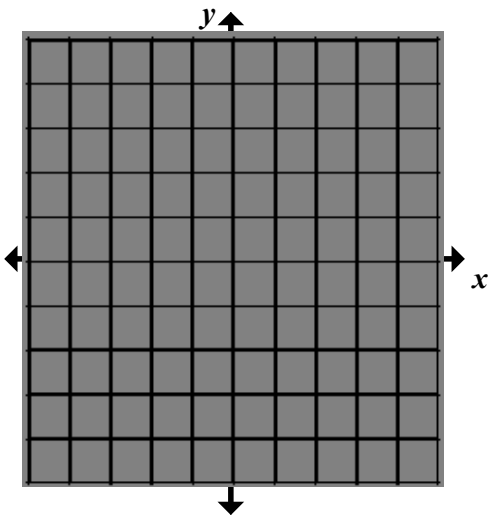
***x* - intercept : _____**

Vertical Asymptote : _____

Horizontal Asymptote : _____

Additional Points :

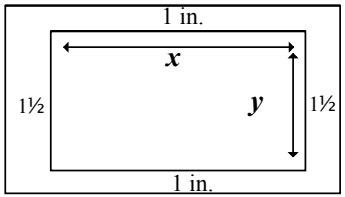
x	*f* (*x*)



See p. 157; exercise 45

Example 6 - Finding a Minimum Area

A rectangular page is designed to contain 48 square inches of print.
The margins on each side of the page are 1½ inches wide.
The margins at the top & bottom are 1 inch deep.
What should the dimensions of the page be so that the minimum amount of paper is used?



Graphical **Numerical**

See p. 158; exercise 65