# 3.1 Exponential Functions & Their Graphs

## What will you learn?

- **To recognize and evaluate exponential functions with base *a***
- **To graph exponential functions**
- **To recognize, evaluate, and graph exponential functions with base *e***
- **To use exponential functions to model & solve real-life problems**

# Exponential Functions

**Nonalgebraic Functions** → **Exponential & Logarithmic**

## Definition of Exponential Function

**The exponential function *f* with base *a* is denoted by**

$$f(x) = ax$$

**Where** $a > 0$

$a \neq 1$

***a* is any real number**

**Think!**

**Why $a \neq 1$ ?**

## Example 1 - Evaluating Exponential Functions

**Use a calculator to evaluate each function at the indicated value of *x***

**a. )** $f(x) = 2x$ $\quad x = -3.1$

**b. )** $f(x) = 2-x$ $\quad x = \pi$

**c. )** $f(x) = 0.6x$ $\quad x = \frac{3}{2}$

**See p. 185; exercise 3**

# Graphs of Exponential Functions

**Graphs of exponential functions have similar characteristics**

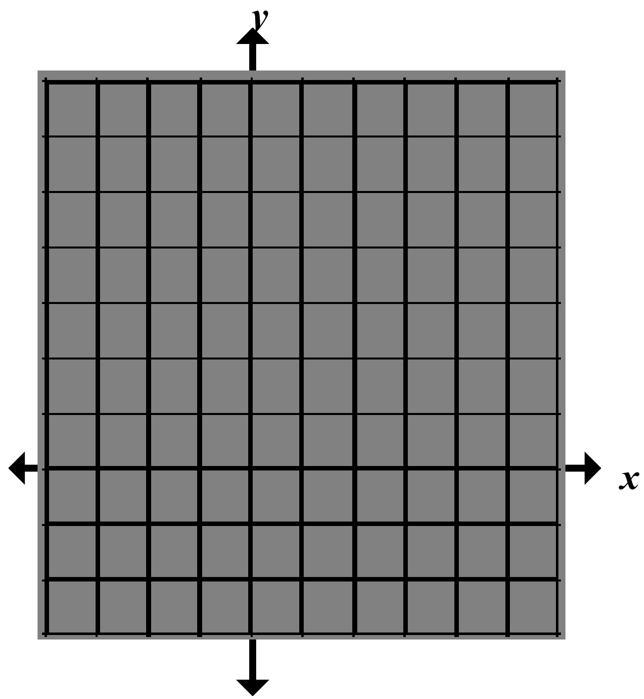
**Example 2 - Graphs of** $y = ax$

**Sketch (by hand) the graphs of the following functions in the same plane**

**a. )** $f(x) = 2x$

**b. )** $g(x) = 4x$

| $x$ | $2x$ | $4x$ |
|---|---|---|
| -2 | | |
| -1 | | |
| 0 | | |
| 1 | | |
| 2 | | |
| 3 | | |




See p. 185; exercise 7

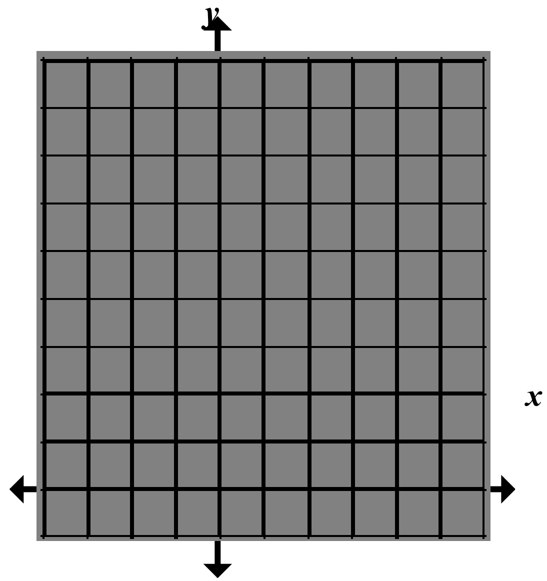
## Example 3 - Graphs of $y = a\text{-}x$

**Sketch (by hand) the graphs of the following functions in the same plane**

**a. )** $F(x) = 2\text{-}x$

**b. )** $G(x) = 4\text{-}x$

| *x* | *2-x* | *4-x* |
|---|---|---|
| *-3* | | |
| *-2* | | |
| *-1* | | |
| *0* | | |
| *1* | | |
| *2* | | |




**See p. 185; exercise 9**

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**THINK!!!**
**Can you write the above functions with POSITIVE EXPONENTS????**

**The properties of exponents can also be applied to real-number exponents!**

$axay =$

$(ab)x =$

$\frac{ax}{ay} =$

$(ax)\,y =$

$a\text{-}x =$

$\left(\frac{a}{b}\right)^{x} =$

$a0 =$

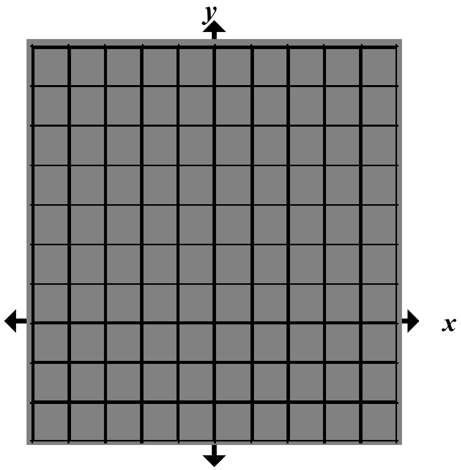
$a2 \quad =$

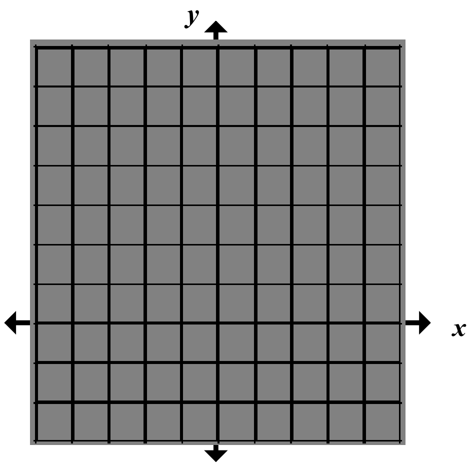
**From examples 2 & 3 - notice**

$F(x) = 2\text{-}x = f(-x)$ $\qquad$ $G(x) = 4\text{-}x = g(-x)$

**What does this tell you about the graphs** $F(x)$ & $f(x)$
$G(x)$ & $g(x)$

**Graph these functions on the same plane using you calculator**




**What are the Domain & Range of the above functions?**

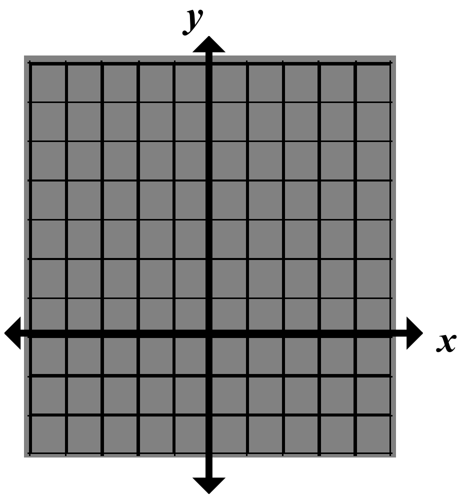
**See Exploration on p. 178**

# Exponential Function

$$f(x) = ax \qquad a > 0, \quad a \neq 1$$

**Distinguishing Characteristic - ________________________________**

$$f(x) = ax \qquad a > 1$$




**Domain:________**

**Range : ________**

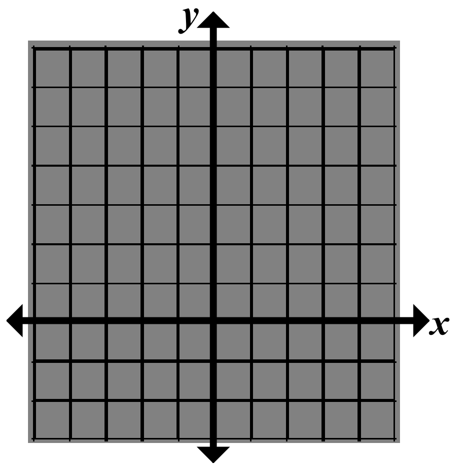
**Intercept: ________**

**Increasing on: _______**

**Horizontal Asymptote: ________**

**Continuous**

$$f(x) = a\text{-}x \qquad a > 1$$




**Domain:________**

**Range : ________**

**Intercept: ________**

**Increasing on: _______**

**Horizontal Asymptote: ________**

**Continuous**

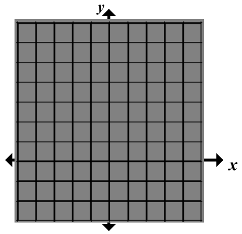
Notice how the graph of $y = ax$ can be used to sketch functions of the form

$$f(x) = b \quad \pm ax + c$$

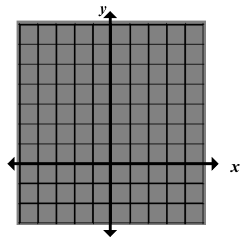
**Example 4 - Transfomations of Graphs of Exponential Functions**

**Each of the following is a transformation of the graph of** $f(x) = 3x$

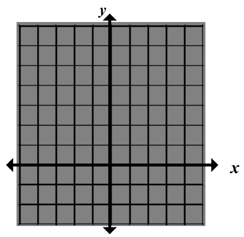
**a.)** $g(x) = 3\ x + 1$



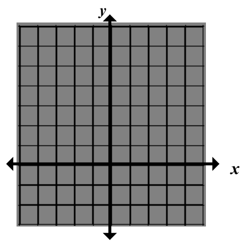

**b.)** $h(x) = 3\ x - 2$




**c.)** $k(x) = -3\ x$




**d.)** $j(x) = 3 - x$




See p. 185; exercise 19

# The Natural Base *e*

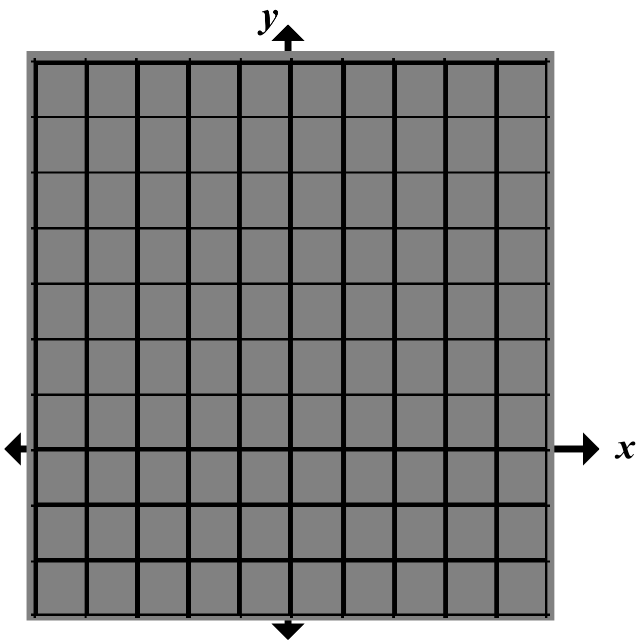
***e*** **is an irrational number**

$$e \approx 2.718281828$$

$f(x) = e x$ → **Natural Exponential Function**

variable

constant



## Example 5 - Approximation of the Number *e*

**Evaluate the expression** $[1 + (1/x)]x$
**for several values of** $x$
**to see that the values approach** $e \approx 2.718281828$
**as** $x$ **increases without bound**

**Graphical**

**Numerical**

See p. 186; exercise 37

**Example 6 - Evaluating the Natural Exponential Function**

**Use a calculator to evaluate the function $f(x) = e^x$ at each value of $x$**

**a.)** $x = -2$

**b.)** $x = 0.25$

**c.)** $x = -0.4$

See p. 186; exercise 23
See Exploration p. 181

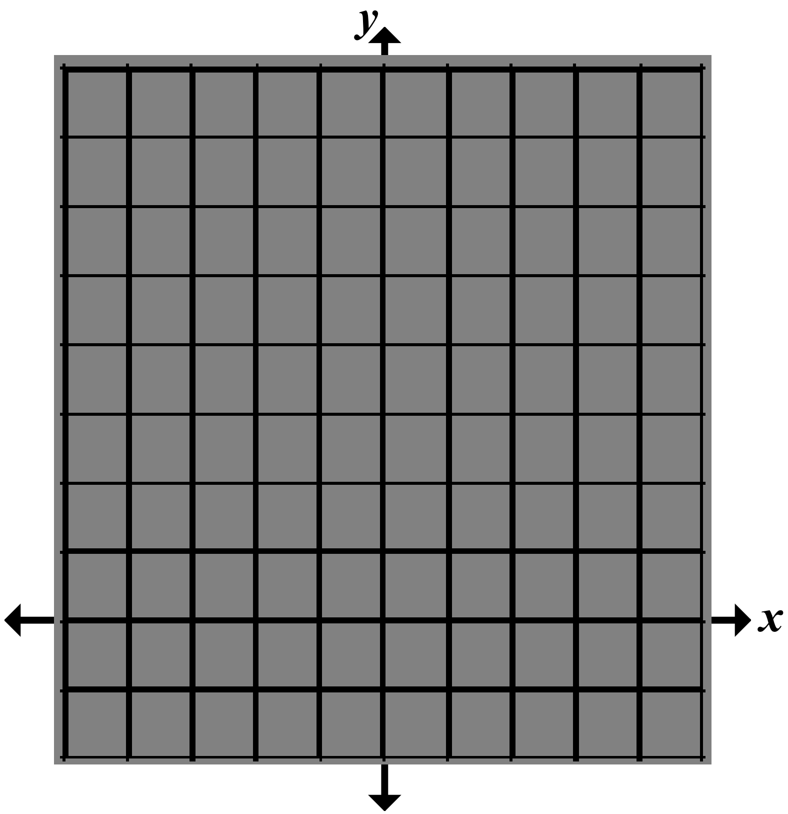
**Example 7 - Graphing Natural Exponential Functions**

**Sketch the graph of each natural exponential function.**

**a. )** $f(x) = 2e^{0.24x}$

**b. )** $g(x) = \frac{1}{2}e^{-0.58x}$

| $x$ | $f(x)$ | $g(x)$ |
|---|---|---|
| -3 | | |
| -2 | | |
| -1 | | |
| 0 | | |
| 1 | | |
| 2 | | |
| 3 | | |




See p. 186; exercise 35

**Applications** **Exponential Growth**

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## Continuously Compounded Interest

Suppose a principal $P$ is invested at an annual interest rate $r$, compounded once a year.
If the interest is added to the principal at the end of the year, the new balance $P1$ is

$$P1 = P + Pr$$
$$= P(1 + r)$$

This pattern is repeated each year.

| Time in Years | Balance after each Compounding |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| ⋮ | |
| t | |

**Quarterly, monthly, daily compoundings**

Let $n$ = # of compoundings /year
$t$ = # of years

$\therefore$ $nt$ = total # times interest is compounded
$r/n$ = interest rate per compounding period

$A = P\left(1 + \frac{r}{n}\right) nt$ account balance after $t$ years

**Continuous Compounding**

After $t$ years, the balance $A$ in an account with principal $P$ and annual interest rate $r$ (decimal) is given by the following formulas

1. for n compounding per year: $A = P\left(1 + \frac{r}{n}\right) nt$

2. Fonr continuous compounding : $A = Pert$

## Example 8 - Finding the Balance for Compound Interest

**A total of $9000 is invested at an annual interest rate of 2.5%, compounded annually. Find the balance after 5 years.**

**Algebraic** **Graphical**

See p. 186; exercise 55

## Example 9 - Finding Compound Interest

**A total of $12,000 is invested at annual interest rate of 3%.**
**Find the balance after 4 years if the interest rate is compounded:**

**a. ) quarterly**

**b. ) continuously**

**See p. 186; exercise 57**

## Activity

1. **Sketch the graphs of the functions**

   $f(x) = ex$ **and** $g(x) = 1 + ex$ **on the same plane.**

2. **Determine the balance** $A$ **ot the end of 20 years if $1500 is invested at 6.5% interest and compounded :**

   **a. ) quarterly**

   **b. ) continuously**

3. **Determine the amount of money that should be invested at 9% compounded monthly to produce a final balance of $30,000 in 15 years.**

**Example 10 - Radioactive Decay**

Let *y* represent a mass of radioactive stontium ( 90Sr), in grams, whose half-life is 28 years.
The quantitiy of strontium present after *t* years is y = 10 ( ½ ) t/28

a. What is the initial mass when t = 0 ?

Algebraic          Graphical

b . How much of the initial mass is present after 80 years?

Algebraic          Graphical

See p. 186; exercise 65

**Example 11 - Population Growth**

**The approximate number of fruit flies in an experimental population after *t* hours is given by $Q(t) = 20e^{0.03t}$, where $t \geq 0$.**

**a.) Find the initial number of fruit flies in the population. ( $t = 0$ )**

**b.) How large is the poplulation of fruit flies after 72 hours?**

**c.) Graph Q**

**See p. 187; exercise 67**