3.2 Logarithmic Functions & Their Graphs

What will you learn?

- To recognize and evaluate logarithmic functions with base *a*
- To graph logarithmic functions
- To recognize, evaluate, and graph natural logaithmic functions
- To use logarithmic functions to model and solve real-world problems

Logarithmic Functions

Exponential Functions pass the horizontal line test $\therefore$ they have inverse functions

The inverse funtion is call the logarithmic function with base a

Definition of Logarithmic Function

For $\left.\begin{array}{l} x > 0 \\ a > 0 \\ a \neq 1 \end{array}\right\}$ $\quad y = log ax \quad$ *iff* $\quad x = ay$

Logarithmic function with base a

$$f(x) = log ax$$

"log base *a* of *x*"

$log ax$ ⟶ **the exponent to which** a **must be raised to obtain** x

Ex) $log\ 28 = 3$ ⟶ $23 = 8$

******A logarithm is an exponent!!******

Example 1 - Evaluating Logarithms

Evaluate each logarithm at the indicated value of x

a.) $f(x) = \log_2 x, \quad x = 32$

b.) $f(x) = \log_3 x, \quad x = 1$

c.) $f(x) = \log_4 x, \quad x = 2$

d.) $f(x) = \log_{10} x, \quad x = \frac{1}{100}$

See p. 195; exercise 17

Logarithmic Functions with base 10 → Common Logarithmic Functions → LOG

Example 2 - Evaluating Common Logarithms on a Calculator

Use a calculator to evaluate the $f(x) = \log_{10} x$ at each value of x

a.) $x = 10$

b.) $x = 2.5$

c.) $x = -2$

d.) $x = \frac{1}{4}$

See p. 195; exercise 21

Properties of Logarithms

1. $\log_a 1 = 0$ → $a^0 = 1$

2. $\log_a a = 1$ → $a^1 = a$

3. $\log_a a^x = x$ and $a^{\log_a x} = x$ **Inverse Property**

4. If $\log_a x = \log_a y$, then $x = y$ **1 : 1 Property**

Example 3 - Using Properties of Logarithms

a.) Solve for x : $\log_2 x = \log_2 3$

b.) Solve for x : $\log_4 4 = x$

c.) Simplify : $\log_5 5^x$

d.) Simplify : $7^{\log_7 14}$

See p. 195; exercise 25

Graphs of Logarithmic Functions

To sketch the graph of $y = logax$

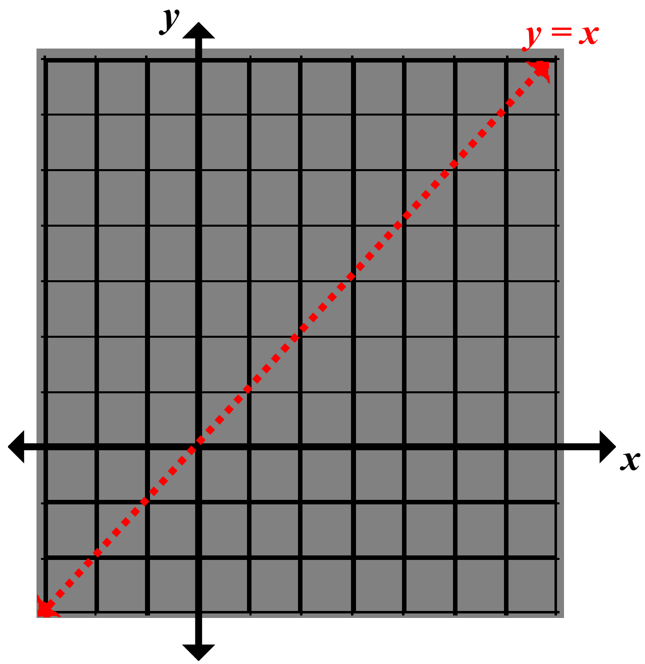
Remember... the graphs of inverse functions are reflections of each other in theline $y = x$

Example 4 - Graphs of Exponential & Logarithmic Functions

In the same coordinate plane, sketch the graph of each function by hand.

a.) $f(x) = 2x$

x	$f(x)$
-2	
-1	
0	
1	
2	
3	



b.) $g(x) = log2x$

See p. 195; exercise 35

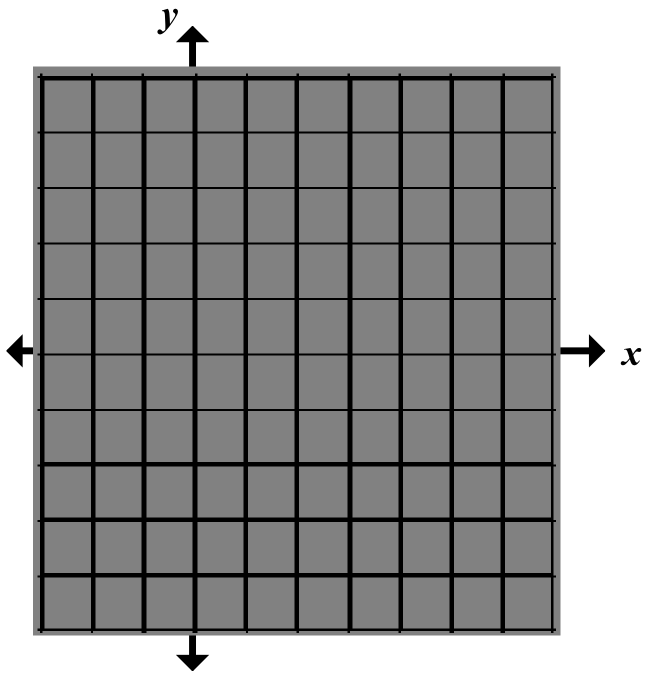
Example 5- Sketching the graph of a Logarithmic Function

$$f(x) = \log_{10} x$$

No calc

x	$f(x)$
1/100	
1/10	
1	
10	
2	
5	
8	

Calc

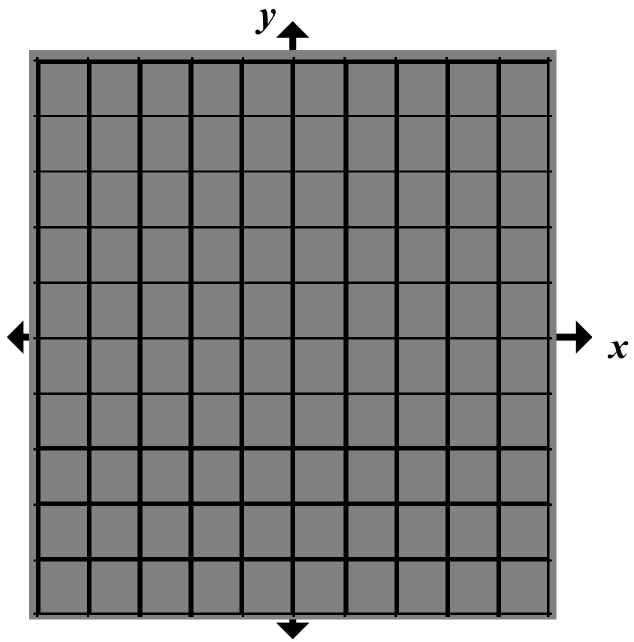


See p. 195; exercise 41

Logarithmic Functions

$$f(x) = \log_a x, \qquad a > 0,\ a \neq 1$$

- **Inverse of the exponential function**
- **Continuous**
- **Reflection of ___________________**



Domain : ______________

Range : ______________

Intercept : _____________

Increasing on : __________

Vertical Asymptote: _____________

Example 6 - Transformations of Graphs of Logarithmic Functions

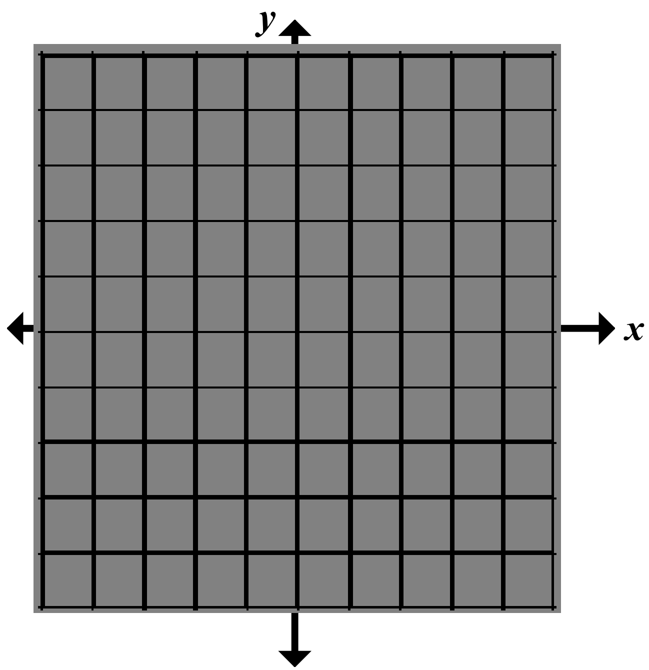
Each of the following is a transformation of the graph of

$f(x) = \log_{10} x$

a.) Because $g(x) = \log_{10}(x - 1)$

$= f(x - 1)$

the graph of *g* can be obtained by shifting the graph of f one unit to the *right*



b.) Because $h(x) = 2 + \log_{10} x$

$= 2 + f(x)$

the graph of *h* can be obtained by shifting the graph two units *upward*

See p. 196; exercise 49

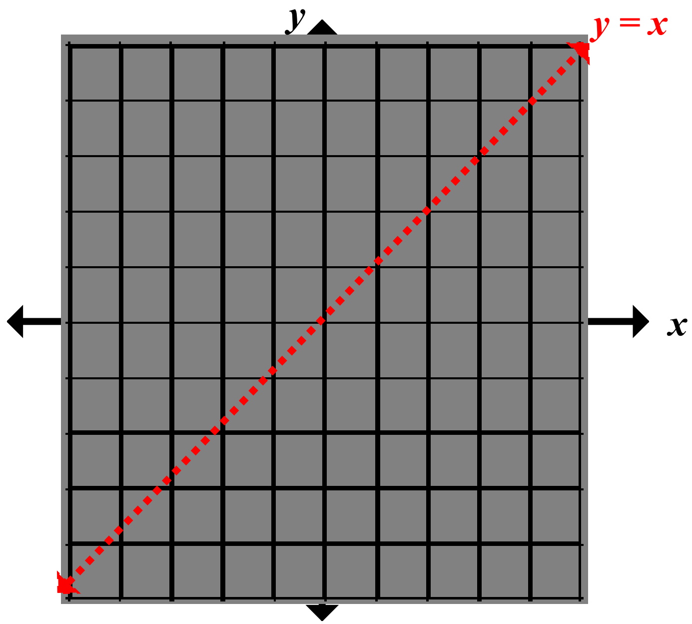
The Natural Logarithmc Function

Inverse of $f(x) = ex$ → $ln\ x$

For $x > 0$ $y = ln\ x$ IFF $x = e\ y$

Natural Logarithmic Function

$$f(x) = log\ ex$$
$$= ln\ x$$



Example 7 - Evaluating the Natural Logarithmic Function

Use a calculator to evaluate $f(x) = \ln x$ at each value of x ln

a.) $x = 2$

b.) $x = 0.3$

c.) $x = -1$

See p. 196; exercise 53

Properties of Natural Logarithms

1. $ln\ 1 = 0$ $e0 = 1$
2. $ln\ e = 1$ $e1 = e$
3. $ln\ ex = x$ and $eln\ x = x$ **Inverse**
4. If $ln\ x = ln\ y$, then $x = y$ **1:1 Property**

Example 8 - Using Properties of Natural Logarithms

a.) $ln\ \frac{1}{e}$

b.) $e\ ln5$

c.) $ln\ e0$

d.) $2\ ln\ e$

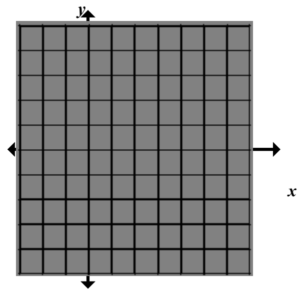
See p. 196; exercise 57

Example 9 - Finding the Domains of Logarithmic Functions

Find the domain of each function:

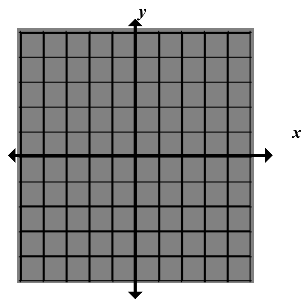
a.) $f(x) = \ln(x - 2)$

Algebraic **Graphical**



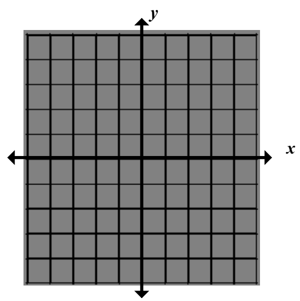
b.) $g(x) = \ln(2 - x)$

Algebraic **Graphical**



c.) $h(x) = \ln x2$

Algebraic **Graphical**



See p. 196; exercise 61

Example 10 - Human Memory Model

Students participating in a psychology experiment attended several lectures on a subject and were given an exam.
Every month for a year after the exam, the students were retested to see how much of the material they remembered.
The average scores fro the group are given by the human memory model

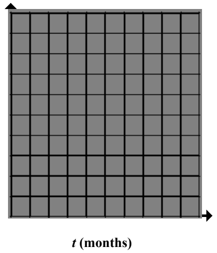
$$f(t) = 75 - 6\ln(t+1), \qquad 0 \le t \le 12$$

where t = time (months)

a.) What was the average score on the original exam ($t = 0$) ?

Algebraic

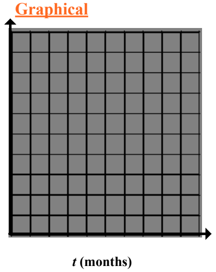
Graphical



b.) What was the average score at the end of $t = 2$ months?

Algebraic

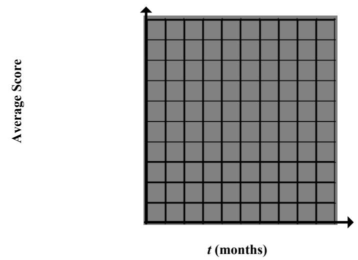
Graphical



c.) What was the average score at the end of $t = 6$ months?

Algebraic

Graphical



See p. 196; exercise 69