3.3 Properties of Logarithms

 **What will you learn?** 

- **To rewrite logarithms with different bases**
- **To use the properties of logarithms to evaluate or rewrite logarithmic expressions**
- **To use the properties of logarithms to expand or condense logarithmic expressions**
- **To use logarithmic functions to model and solve real-life problems**

Change of Base

Most calculators have only 2 types of keys
-Common Logs (Base *10*)
-Natural Logs (Base *e*)

You may occasionally have to evaluate logs with other bases

Change-of-Base Formula

Let *a*, *b* and *x* be positive real numbers s.t. $a \neq 1$ and $b \neq 1$.
Then $\log_a x$ can be converted to a different base using any of the following formulas

Base *b*

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Base *10*

$$\log_a x = \frac{\log_{10} x}{\log_{10} a}$$

Base *e*

$$\log_a x = \frac{\ln x}{\ln a}$$

One way to look at it...
logs to base *a* are constant multiples of logs to base *b* → $\frac{1}{\log_b a}$

Example 1- Changing Bases Using Common Logs

a.) $\log_4 25$

b.) $\log_2 12$

See p. 203; exercise 9

Example 2 - Changing Bases Using Natural Logs

a.) $\log_4 25$

b.) $\log_2 12$

See p. 203; exercise 11

W THESE PROPERTIES!!!

Properties of Logarithms

Let a be a positive number s.t. $a \neq 1$.
Let n be a real number.
Let u and v be positive real numbers.

Logs with Base a	Natural Logs
1. $\log_a uv = \log_a u + \log_a v$	1. $\ln uv = \ln u + \ln v$
2. $\log_a \frac{u}{v} = \log_a u - \log_a v$	2. $\ln \frac{u}{v} = \ln u - \ln v$
3. $\log_a u^n = n \log_a u$	3. $\ln u^n = n \ln u$

Example 3 - Using Properties of Logs

Write each logarithm in terms of $\ln 2$ and $\ln 3$

a.) $\ln 6$

b.) $\ln \frac{2}{27}$

See p. 203; exercise 19

Example 4 - Using Properties of Logs

Use the properties of logarithms to verify that

$$log10\frac{1}{100} = log\,10100$$

See p. 203; exercise 21

Rewriting Logarithmic Expressions

Example 5 - Expanding Logarithmic Expressions

Use the properties of logs to expand each expression.

a.) $\log_4 5x^3y$

b.) $\ln \sqrt{\frac{3x - 5}{7}}$

See p. 203; exercise 39

Example 6 - Condensing Logarithmic Expressions

Use the properties of logs to condense each logarithmic expression.

a.) $\frac{1}{2} \log_{10} x + 3 \log_{10}(x+1)$

b.) $2 \ln(x+2) - \ln x$

c.) $\frac{1}{3}[\log_2 x + \log_2(x-4)]$

See p. 204; exercise 57

Example 7 - Finding a Mathematical Model

See p. 202

See p. 204; exercise 83