3.4 Solving Exponential & Logarithmic Equations

What will you learn?

- To solve simple exponential & logarithmic equations
- To solve more complicated exponential equations
- To solve more complicated logarithmic equations
- To use exponential & logarithmic equations to model & solve real-world problems

2 Basic Strategies

$a > 0$ $\quad$ $a \neq 1$

One-to -One Properties	Inverse Properties
$a^x = a^y$ iff $x = y$	$a^{\log_a x} = x$
$\log_a x = \log_a y$ iff $x = y$	$\log_a a^x = x$

Example 1 - Solving Simple Exponential & Logarithmic Equations

Original Equation	Rewritten Equation	Solution	Property
a.) $2^x = 32$			
b.) $\ln x - \ln 3 = 0$			
c.) $(1/3)^x = 9$			
d.) $e^x = 7$			
e.) $\ln x = -3$			
f.) $\log_{10} x = -1$			

See p.213; exercise 21

Strategies for Soling Exponential & Logarithmic Equations

1. Rewrite - allow use of 1:1 Property

2. Rewrite - *Exponential* → *Logarithmic* Use Inverse Property of Logs

3. Rewrite - *Logarithmic* → *Exponential* Use Inverse Property of Exponentials

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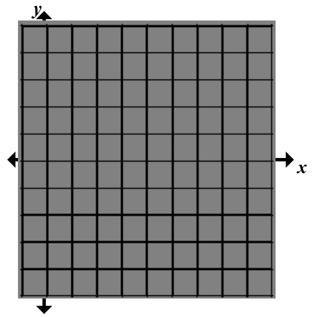
Example 2 - Solving Exponential Equations

Solve each equation.

a.) $e\ x = 72$

Algebraic

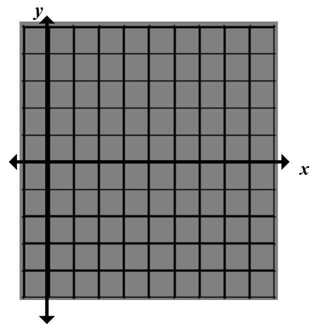
Graphical



b.) $3\ (2\ x) = 42$

Algebraic

Graphical



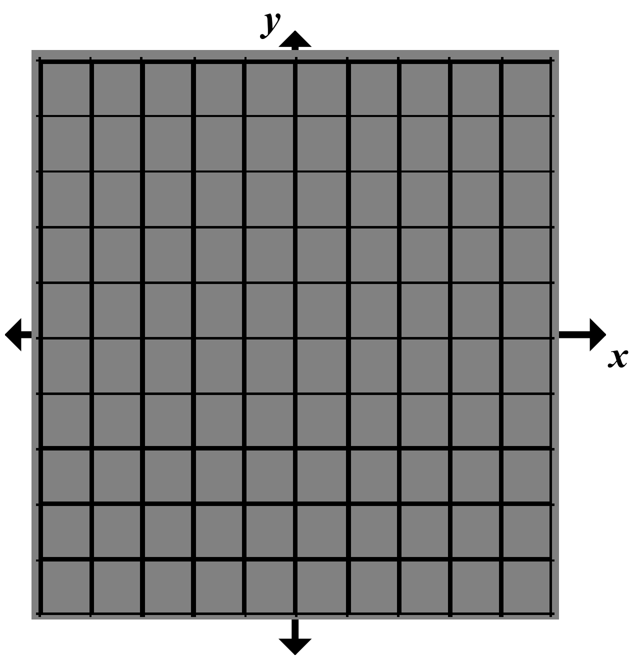
See p. 213 ; exercise 45

Example 3 - Solving Exponential Equations

Solve $4e^{2x} - 3 = 2$

Algebraic

Graphical



See p. 213; exercise 49

Example 4 - Solving an Exponential Equation

Solve $2(3^{2t-5}) - 4 = 11$

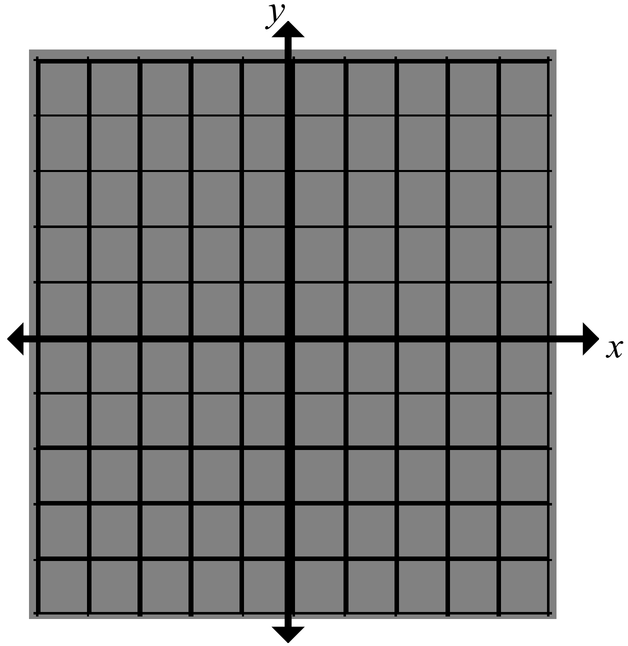
See p.213; exercise 53

Example 5 - Solving an Exponential Equation in Quadratic Form

Solve $e^{2x} - 3e^{x} + 2 = 0$

Algebraic

Graphical



See p. 213; exercise 55

Solving *Log* Equations

Exponentiate each side

Example

$ln\ x = 3$ — *Log* Form

$e^{ln\ x} = e^3$ — *Exponentiate* each side

$x = e^3$ — Exponential Form

Example 6 - Solving *Log* Equations

a.) $ln\ x = 2$

b.) $log_3(5x - 1) = log_3(x + 7)$

See p. 214; exercise 77

Example 7 - Solving *Log* Equations

Solve $5 + 2 \ln x = 4$

Algebraic

Graphical

See p. 214; exercise 77

Example 8 - Solving a *Log* Equation

Solve $2 \log 53x = 4$

See p. 214; exercise 81

Example 9 - Checking for Extraneous Solutions

Solve $ln\,(x - 2) + ln\,(2x - 3) = 2\,ln\,x$

Algebraic

Graphical

CHECK !

See p. 214; exercise 89

Example 10 - The Change-of-Base-Formula

Prove the change-of-base formula:

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Example 11 - Approximating the Solution of an Equation

Approximate (to 3 decimals) the solution of :

$$\ln x = x^2 - 2$$

See p. 215; exercise 97

Example 12 - Doubling an Investment

$$$$

You have deposited $500 in an account that pays 6.75% interest, compounded continuously. How long will it take for your money to double?

See p. 215; exercise 109

Example **$$$$**

A deposit of $5000 is placed in a savings account for 2 years.
The interest is compounded continuously.
At the end of 2 years, the balance is $5416.50.
What is the annual interest rate for this account?

Example 13 - Average Salary for Public School Teachers

$$$$

For selected years from 1980-2000, the average salary (thousands of $) for public school teachers for the year *t* can be modeled by the equation :

$$y = -39.2 + 23.64 \ln t, \qquad 10 \le t \le 30$$

$t = 10$ **represents 1980**

During which year did the average salary for public school teachers reach $40.0thousand?

(NEA)

See p. 215; exercise 118