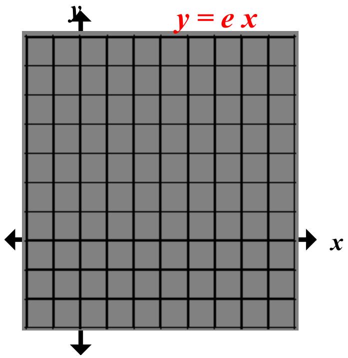
3.5 Exponential and Logarithmic Models

5 Most Common Types of Exponential & Logarithmic Models

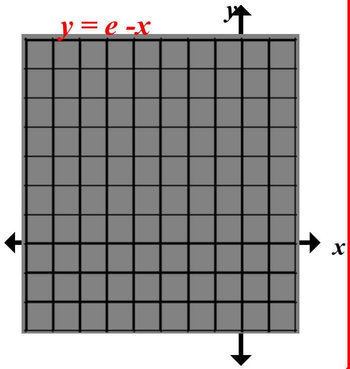
Exponential Growth

$y = ae^{bx}, \quad b > 0$



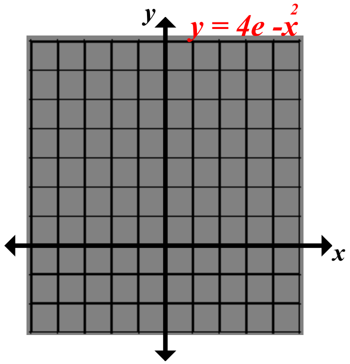
Exponential Decay

$y = ae^{-bx}, \quad b > 0$



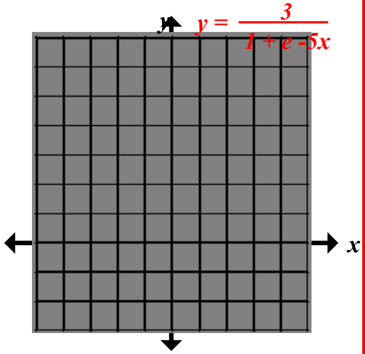
Gaussian Model

$y = ae^{-(x-b)^2/c}$



Logistic Growth Model

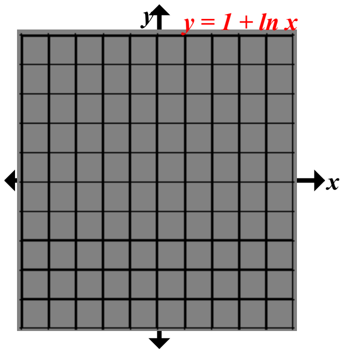
$y = \dfrac{a}{1 + be^{-rx}}$



Logarithmic Models

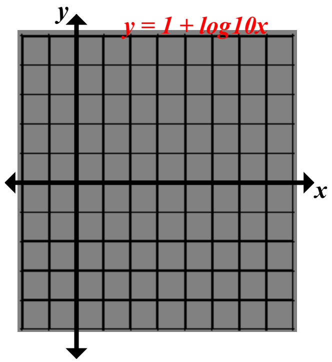
Natural Log

$y = a + b \ln x$



Common Log

$y = a + b \log_{10} x$



Exponential Growth & Decay

Example 1 - Population Growth

See p. 218

See p. 225; exercise 27

Example 2 - Modeling Population Growth

In a research experiment, a population of fruit flies is increasing according to the law of exponential growth.
After 2 days there are 100 flies.
After 4 days there are 300 flies.
How many flies will there be after 5 days?

See p. 225; exercise 29

See p. 220

Carbon Dating Model $R = \frac{1}{1012} e^{-t/8267}$

Example 3 - Carbon Dating

The ratio of carbon 14 to carbon 12 in a newly discovered fossil is $R = \frac{1}{1013}$
Estimate the age of the fossil.

Algebraic

Graphical

See p. 226; exercise 32

Gaussian Models

$$y = ae^{-(x-b)^2/c}$$

- Commonly used in probability & stats
- Represents poulations that are normally distributed
- Bell -Shaped Curve

Standard Normal Distribution

$$y = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

Example 4 - SAT Scores

In 2002, SAT math scores roughly followed the normal distribution

$$y = 0.0035e^{-(x-516)^2/25{,}992} \quad 200 \le x \le 800$$

where x = SAT math score
Use a calculator to graph and estimate the average SAT score

College Board

See p. 226; exercise 37

Logistic Growth Models

$$y = \frac{a}{1 + be^{-rx}}$$

y = population size
x = time

Example 5 - Spread of a Virus

On a college campus of 5000 students, one student returns from a vacation with a contagious flu virus. The spread of ths virus is modeled by

$$y = \frac{5000}{1 + 4999\,e^{-0.8t}} \qquad t \geq 0$$

y = total # infected after *t* days

The college will cancel classes when 40% or more of the students are infected.
How many students are infected after 5 days?
After how any days will the college cancel classes?

Algebraic | **Graphical**

See p. 226; exercise 39

Logarithmic Models

Richter Scale - measures the intesity of an earthquake

$$R = log\,10 \frac{1}{10}$$

R **= magnitude**
I **= intensity**

Example 6 - Magnitude of Earthquakes

**In 2001, the coast of Peru experienced an earthquake that measured 8.4 on the Richter Scale.
In 2003, Colima, Mexico experienced and earthquake that measured 8.6 on the Richter Scale.
Find th intensity of each earthquake and compare the two intensities.**

See p. 227; exercise 41