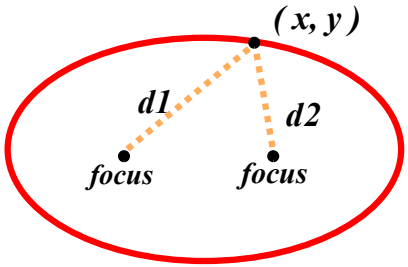
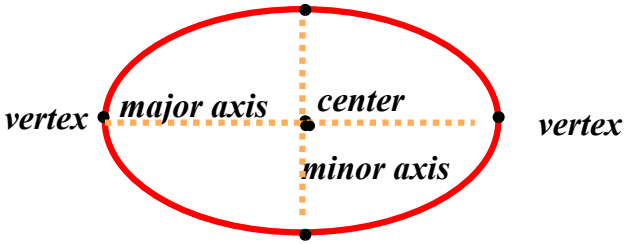
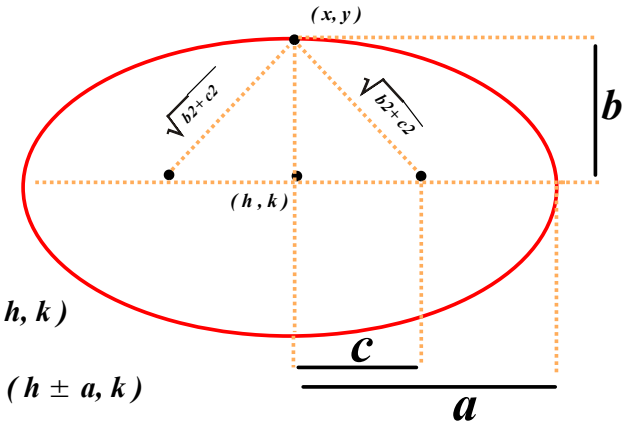
Definition of Ellipse

An ellipse is the set of all points (x, y) in a plane, the sum of whose distances from two distinct fixed points (foci) is constant.





- **The line *through the foci* intersects the ellipse at 2 points called the vertices.**
- **The *chord joining the vertices* is the major axis.**
- **The *midpoint of the chord* is the center.**
- **The *chord perpendicular to the major axis at the center* is the minor axis.**



Center : (h, k)

Vertices : $(h \pm a, k)$

Foci: $(h \pm c, k)$

$$2\sqrt{b2+c2} = 2a$$

$$b2+c2 = a2$$

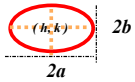
Length of the major axis

$(a + c) + (a - c) = 2a$

Standard Equation of an Ellipse

The *standard equation of an ellipse* the center (h, k) and major and minor axis of lengths $2a$ and $2b$, respectively, where $0 < b < a$, is

$$\frac{(x-h)2}{a2} + \frac{(y-k)2}{b2} = 1$$ **Major Axis is Horizontal**



$$\frac{(x-h)2}{b2} + \frac{(y-k)2}{a2} = 1$$ **Major Axis is Vertical**

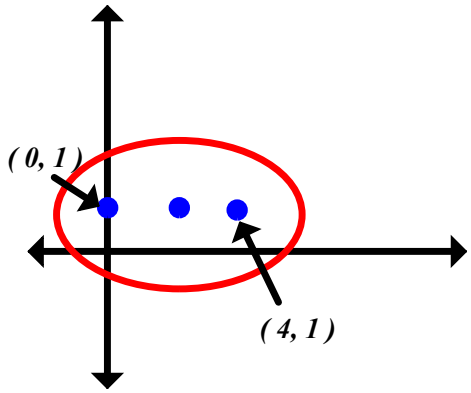
The foci lie on the major axis, c units from the center, with $c2 = a2 - b2$.
If the *center is at the origin* $(0, 0)$, the equations takes one of the following forms:

$$\frac{x2}{a2} + \frac{y2}{b2} = 1$$ **Major Axis is Horizontal**

$$\frac{x2}{b2} + \frac{y2}{a2} = 1$$ **Major Axis is Vertical**

Example 1 - Finding the Standard Equation of an Ellipse

Foci : $(0, 1)$ **and** $(4, 1)$



Solution

Find : Center _____

$c =$ _____

(Distance form Center to on Foci)

$a =$ _____

$b =$ _____

Major Axis : Horizontal Vertical

Equation:

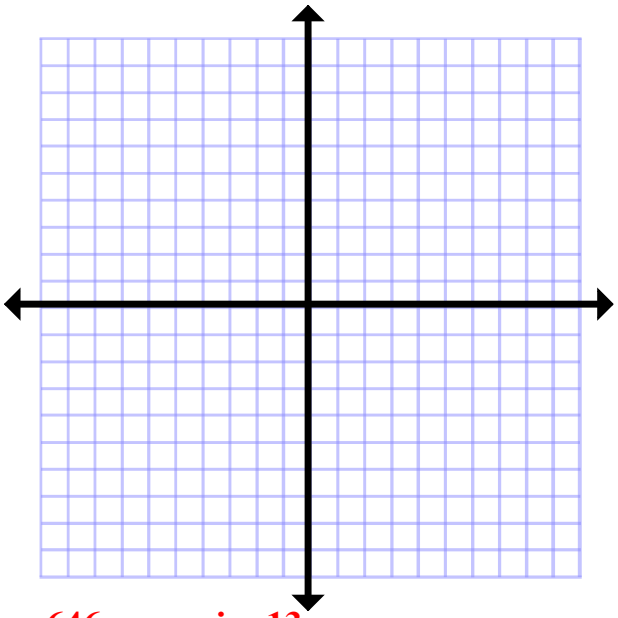
See p. 647; exercise 35

Example 2 - Sketching an Ellipse

Sketch the ellipse given by : $4x2 + y2 = 36$

Algebraic

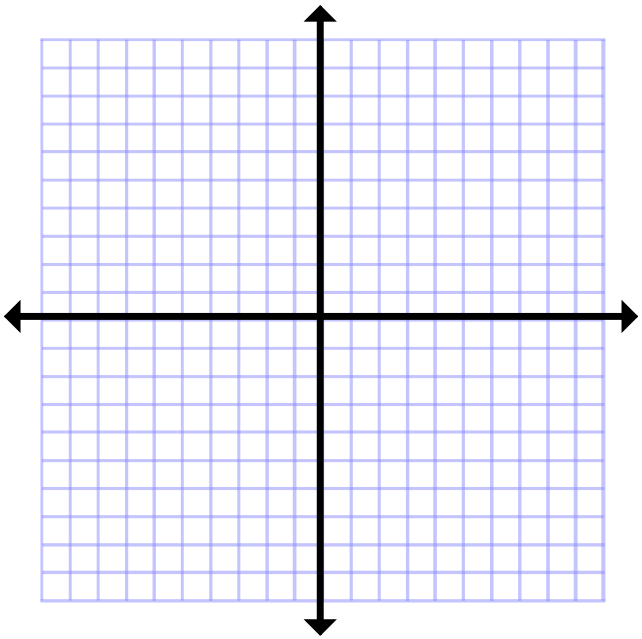
Graphical



See p. 646; exercise 13

Example 3 - Sketching an Ellipse

Sketch the graph of the ellipse given by : $x2+4y2+6x-8y+9=0$



See p. 646; exercise 15

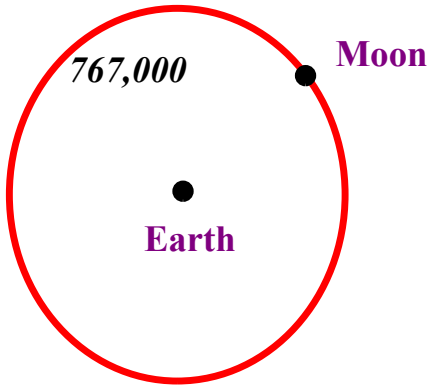
Example 4 - Analyzing an Ellipse

Find the center, vertices, and foci of the eliipse: $4x2+y2-8x+4y-8=0$

Complete the square to write the equation in standard form

Example 5 - An Application using Elliptical Orbit

The moon travels about Earth in an elliptical orbit with Earth at one focus, as shown. The major and minor axes of the orbit have lengths of 768,000 km and 767,640 km, respectively.Find the greatest and smallest distances (*the apogee and perigee*) from Earth's center to the moon's center.



Eccentricity

The measure of the ovalness. When the foci of the ellipse are close to the center it is difficult to determine tha the shape is an oval.

The eccentricity of an ellipse is given by: $e = \frac{c}{a}$