Density: Deriving a Mathematical Relationship

Background: Density is the physical property that compares the mass of a substance to the volume it occupies. Density values for solids and liquids are typically expressed in terms of the amount of substance contained in 1 cm^3 (or 1 ml) of space. The density of a liquid, a solid, or a gas may change with changes in temperature or pressure. When most liquids and solids warm up, they expand slightly, making their volumes greater. However, mass does not change, so the density value decreases. (Changes in pressure do not noticeably affect liquids or solids.) The effect of these changes on the densities of gases is much more significant because the volume occupied by a gas is mostly "empty" space, and the particles themselves can spread out or condense more freely. For this reason, the density of a gas is usually expressed in terms of a larger volume (1 L). The container's entire volume is considered to be the volume of the gas. Published density values always indicate the temperature and pressure at which the value was obtained.

Procedure/Data: Collect and record mass and volume data for two substances, copper and aluminum.

a) For mass, obtain the mass of a small container filled with the sample of metal. After emptying the container into water as described in (b) below, obtain the mass of the container alone. Subtract these masses to obtain the mass of the sample itself. Be sure to record ***each*** of these masses on a data chart that you have designed. Identify the data clearly and use unit labels.
b) For volume, use water displacement. It is known that an object placed in water will displace an amount of water equal to its own volume. Fill a large graduated cylinder about half-way with tap water. Record the volume to the nearest 0.1 ml. Always read the bottom of the liquid curve (the "meniscus"). Carefully add the sample to the water and tap out air bubbles. Record the new volume. Subtract these volumes to obtain the volume of the sample itself. Record all values.
c) Drain the water off of each sample and spill it onto a separate piece of paper towel to dry.
d) Record the mass and volume of your sample on the board so others can copy it.

Analysis: Use class data to graph the relationship between mass and volume for copper and for aluminum. In this case, mass is the dependent variable, (y-axis) and volume is the independent variable (x-axis). In other words, you are designing the graph to show how mass responds to changes in volume.

Each of the corresponding mass and volume values forms an ordered pair, and should be graphed as a single point. Do not connect the points – instead, we will look for a trend.

Your class will use graphing software to complete the data analysis. In the event that you must do this individually, and do not have access to graphing software, here is information about graphing manually. Either way, it is important that you know how to apply the equation for a straight line.

Use a straight-edge to draw the best-fit straight line through each set of plotted points. Use your judgment here - all data points may not fall exactly on a line, but you should be able to find a trend through which you can draw a straight line. This process is known as "curve-fitting", and should include the majority of the points from data. Be sure to include the point **(0, 0)** on this best-fit line, because it is obvious that the volume is 0 ml if the mass is 0 grams. Determine the slope of each line. Show the calculations neatly with the graph. Recall "rise over run" method as well as the equation, "$y = \mathbf{m}x + b$", for calculating slope. Be able to apply both of these slope calculation methods.

Error: Calculate the percent error of the densities you obtained. Use the slope value as the experimental value. Look up the accepted values in the reference section of your textbook.

Conclusion:

1. What does this activity demonstrate about the relationship between the mass and volume of a substance? Is density a characteristic (identifying) property of a substance?
2. How are the following related: density, slope, **m** (in $y = mx + b$)?
3. The experimental determination of density by the slope method is generally more accurate than by using a single value for mass and volume solving for "M/V". Explain why this is so.

Problems: Density and related problems

Use dimensional analysis to solve the following problems on loose-leaf.

- ✓ Be sure to use unit labels throughout the problem.
- ✓ You should round off answers to the correct number of significant figures – consider only measured values, not well-known definitions, "constants", or facts.
- ✓ If you are having difficulty with unit conversions, refer to your notes for prefix definitions.
- ✓ **Remember that "per" means "equals", and that densities are expressed in terms of 1 cm^3 or 1 ml. Therefore, a density of 0.680 g/cm^3 should be thought of as "0.680 g = 1 cm^3".** When you use a density term as a conversion factor, write it to have a true numerator and a true denominator. You need to think of the "/" as "over". So, for a density of 0.680g/cm^3…

  …write $\frac{\mathbf{0.680\ g}}{\mathbf{1\ cm^3}}$ or $\frac{\mathbf{1\ cm^3}}{\mathbf{0.680\ g}}$ (depending on what you want to cancel)

1. An unknown liquid has a mass of 30.6 g and a volume of 52.3 ml. What is the density of the liquid?

2. The density of gold is 19.3 g/cm^3. What is the mass of 11.3 cm^3 of gold?.

3. A chain bracelet is found. It looks as though it is made either of gold or of copper. The bracelet has a mass of 45.00 g. When it is submerged into a graduated cylinder of water, the volume of the water rises from 9.2 ml to 11.7 ml. Is the bracelet more likely to be gold or copper? Explain

4. The density of silver is 10.5 g/cm^3. What is the volume of a piece of silver having a mass of 31.50 g?

5. The density of ice is 0.917 g/cm^3. What is the volume of 52.35 g of ice? (Why does ice float on water?)

6. A cube of aluminum has a mass of 42.18 grams. The density of aluminum is 2.7 g/cm^3. Determine the volume of the cube. What is the length of each side?

7. The density of gold is 19.3 g/cm^3. The density of sand is 2.5 g/cm^3. What volume of sand would have the same mass as a gold icon with a volume of 1.0 dm^3? (Have you ever seen the movie, Indiana Jones and Raiders of the Lost Ark? This problem describes Indy's mistake!)

8. The density of gold is 19.3 g/cm^3. 196.97 grams of gold contain 6.02×10^{23} atoms of gold. (Chemists call this quantity "1 mole".) What is the mass of 1 atom of gold? What is the volume of 1 atom of gold?

9. What is the mass, in kilograms, of 14.0 L of gasoline? The density of gasoline is 0.680 g/cm^3.