

$$2y^3 + 6x^2y - 12x^2 + 6y = 1$$

A.) Show that  $y' = \frac{4x - 2xy}{x^2 + y^2 + 1}$

$$6y^2 y' + (6x^2 y' + 12xy) - 24x + 6y' = 0$$

$$(6y^2 + 6x^2 + 6) y' = 24x - 12xy$$

$$y' = \frac{24x - 12xy}{6y^2 + 6x^2 + 6}$$

$$y' = \frac{4x - 2xy}{y^2 + x^2 + 1}$$

B.) Write an equation of each horizontal tangent line to the curve.

Tangent line is horizontal iff  $y' = 0$

( $4x - 2xy = 0$  and  $y^2 + x^2 + 1 \neq 0$ )

$$4x - 2xy = 0$$

$$2x(2 - y) = 0$$

$\therefore x = 0$  or  $y = 2$   $y = 2$  is a HA  $\therefore$  there is no value for  $y = 2$

$$x = 0 \rightarrow 2y^3 + 6y - 1 = 0$$

There is exactly one solution - graph or Des Cartes Rule of Signs

Using Calculator ...  $x = 0$   $y = .165$

*The equation of the horizontal tangent line is  $y = .165$*

C.) The line through the origin with slope = -1 is tangent to the curve at point P.

Find the x and y coordinates of point P

Equation of the line  $y = -x$

At the point of tangency:  $\frac{4x - 2xy}{y^2 + x^2 + 1} = -1$  and  $y = -x$

$$4x^2 + 4x + 1 = 0$$

$$x = \frac{-1}{2} \longrightarrow y = \frac{1}{2} \longrightarrow P\left(\frac{-1}{2}, \frac{1}{2}\right)$$