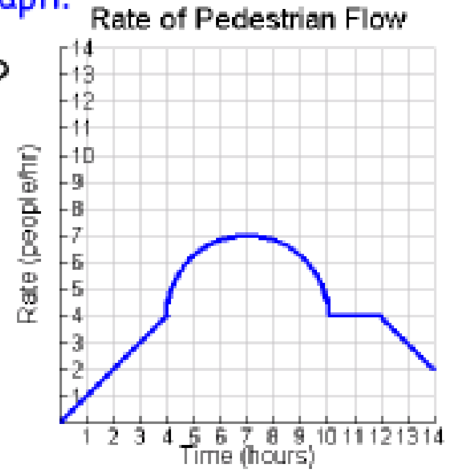
Applications of the First Fundamental Theorem of Calculus

The volume of foot traffic crossing a bridge is depicted on the graph.



1. At what rate are people crossing the bridge when $t = 11$ hours?

2. At what rate is the rate of foot traffic changing when $t = 13$ hours? Justify.

3. Find the total number of people who crossed the bridge on the following intervals:

   a. (0, 4) hrs

   b. (4, 10) hrs

   c. (10, 12) hrs

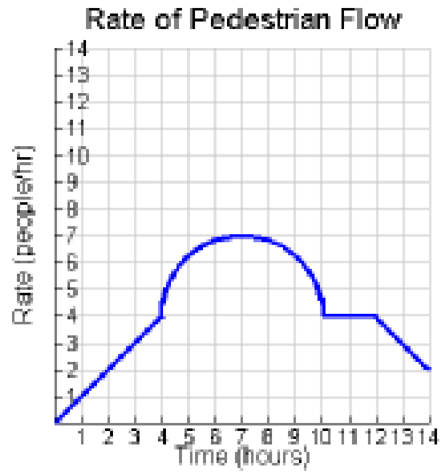
   d. (12, 14) hrs

4. What is the average foot traffic over the entire 14 hrs?

Let's look at how this applies to our concept of integrals.

If you find the area under the graph by rectangles, what units do you end up with?

So you could say that integrating removes the _______________.



The First Fundamental Theorem of Calculus can also be called the "Net Change" Theorem and has many applications to problem situations.

Let's look at some situations and see what happens when we integrate.

1. If V(t) is the volume of water in a reservoir at time, t. It's derivative V'(t) is the rate at which water flows in the reservoir at time t.

So. . . $\int_{t_1}^{t_2} V'(t)dt =$ ____________ which means ____.

2. The rate of growth of a population is g'(t).

So. . . $\int_{t_1}^{t_2} g'(t)dt =$ ____________ which means ____.

3. If an object moves along a straight line with position function, s(t), then it's velocity is s'(t).

So. . . $\int_{t_1}^{t_2} v(t)dt =$ ____________ which means ____.

4. The rate of growth of a child in pounds per year is w'(t).

So. . . $\int_{5}^{10} w'(t)dt =$ ____________ which means ____.

So how are we going to use this? Let's see.

The rate of consumption of oil in the US during the 1980s (in billions of barrels per year) is modeled by the function $C(t)$ below, where t is the number of years after Jan. 1, 1980. Find the consumption of oil in the US from Jan 1, 1980 to Jan 1, 1990.

$$C(t) = 27.08e^{t/25}$$

There is another way to use the FTC to our advantage. Look at the example below.

A honeybee population starts with 100 bees and increases at a rate of n'(t) bees per week.

What does $100 + \int_0^{15} n'(t)\,dt$ represent? ________________________________

Where is the 100 coming from? ________________________________

What does the integral give you? ________________________________

$$f(b) = f(a) + \int_a^b f'(x)\,dx$$

Want = Have + integral from have to want

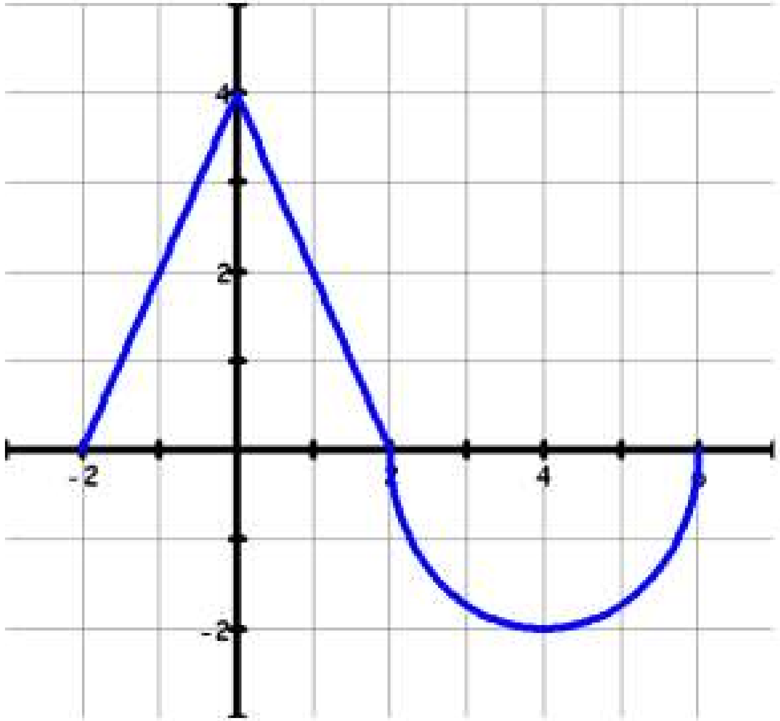
Water flows into a tank at a rate of $\frac{dW}{dt} = \frac{1}{75}\left(600 + 20t - t^2\right)$ where $\frac{dW}{dt}$ is measured in gallons per hour and t is measured in hours. If there are 150 gallons of water in the tank at time $t=0$, how many gallons of water are in the tank when $t=24$ hours?

A particle travels with velocity, $v(t) = (t-2)\sin t$ m/sec for $0 \le t \le 4$ sec. If at time $t=1$ second, the particle is at 1 meter, where is the particle at $t=4$ seconds?

If $y' = 3x^2 + 4x - 5$ and $y(2) = -1$, find $y(3)$.

The graph of f' on [-2, 6] consists of two line segments and a semicircle as shown.

Given that $f(-2)=5$, find:



$f(0)$

$f(2)$

$f(6)$