Accumulating Rates of Change to Find Total Amounts

Last class we looked at the integral as a net change function. We will look at more involved problems today, but one thing needs to be kept in mind...whaterver applications are chosen, the emphasis is on **using the integral of a rate of change to give accumulated change.**

Keep this in mind as we do these problems. I cannot show you every situation that comes up, but I can be sure that you will see these problems over and over again this year.

Shall we begin?

Let's start easy...

The rate at which the world's oil is being consumed is continuously increasing. Suppose the rate (in billions of barrels per year) is given by the function $r = 32e^{0.05t}$ where t is measured in years and t=0 is the start of 1990.

What is the total quantity of oil used between the start of 1990 and the start of 1995?

For $0 \le t \le 31$, the rate of change of the number of mosquitos on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitos per day. There are 1000 mosquitos on Tropical Island at time $t = 0$.

(a) Show that the number of mosquitos is increasing at time $t = 6$.

(b) At time $t = 6$, is the number of mosquitos increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

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(c) According to the model, how many mosquitoes will be on the island at time $t = 31$. Round your answer to the nearest whole number.

(d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \le t \le 31$? Show the analysis that leads to your conclusion.

The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate

$$P'(t) = 1 - 3e^{-0.2\sqrt{t}} \text{ gallons per day,}$$

where t is measured in days. There are 50 gallons of the pollutant in the lake at time t=0. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

(a) Is the amount of pollutant increasing at time t=9? Why or why not?

(b) For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.

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(c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.

(d) An investigator uses the tangent line approximation to P(t) at t=0 as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?

Sand is being poured into a bin that is initially empty. During the work day, for $0 \le t < 9$ hours, the sand pours into the bin at the rate given by

$$S(t) = \frac{5000}{t^3 + 50} \text{ cubic meters per hour.}$$

After one hour, for $1 \le t < 9$, sand is removed from the bin at the rate of

$$R(t) = 23.967\sqrt{t} \text{ cubic meters per hour.}$$

(a) How much sand is put into the bin during the work day? Indicate units of measure.

(b) Find S(6) - R(6); include units of measure. Explain what this number means in the context of the problem?

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(c) How much sand is in the bin at the end of the work day?

(d) What is the maximum amount of sand in the bin for the work day, $0 \le t < 9$, and when does it occur? Justify your answers.

Rivers and streams flow into Lake Buchanan and heavy rains can cause flooding. The LCRA starts monitoring the level of water in the lake when heavy rains start and will open floodgates on the Lake Buchanan dam to allow water to flow downstream to minimize flood damage.

Let $E(t) = t + \sin t$ hundred cubic ft per hour be the rate of water entering Lake Buchanan.

Let $R(t) = \begin{cases} 2 & 1 \le t < 6 \\ 10.5 & 6 < t \le 12 \end{cases}$ hundred cubic ft per hour be the rate of water being released.

(a) What is the average rate of change of the water entering the lake on $[0, \pi]$? For what value of t does the instantaneous rate of change equal this?

(b) How much water has entered the lake in the first hour of monitoring?

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(c)At what time t is the rate of water entering the lake equal to the rate of the water being released?

(d) From [2, 12] hours, give the time intervals when more water was entering the lake than was being released. Determine how much more water entered in each interval.

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(e) If A represents the amount of water in the lake at time t=0, set up an integral to represent the total amount of water in the lake at time t=7.

(f) At what time t will the amount of water in the lake return to the amount of water at time t=0?