Functions Defined by Integrals

Let's look at the function $F(x)=\int_0^x f(t)dt$

What does this mean geometrically?

We often call this function the accumulation function? Can you guess why?

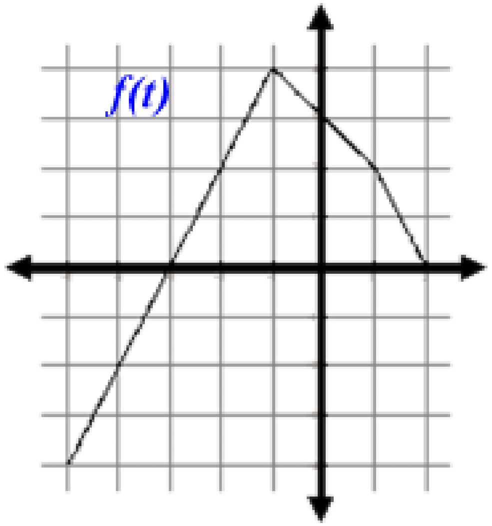
Is there a relationship between $f(t)$ and $F(x)$?

Since we now know that f is really the derivative of F(x), let's review a little curve sketching.

- f(x) is increasing when f'(x) is ______________
- f(x) is decreasing when f'(x) is ______________
- f(x) has a max when f'(x) __________________
- f(x) has a min when f'(x) ___________________
- f(x) is concave down when f''(x) is __________________
- f(x) is concave up when f''(x) is _____________________
- f(x) is concave down when f'(x) is ____________________
- f(x) is convave up when f'(x) is ______________________
- f(x) has a point of inflection when f'(x) has __________________

Let's see how to apply this to our concept of the accumulation function.

The graph of the function, f(t), is given. $g(x) = \int_{-1}^{x} f(t)dt$



1. Find g(-5) and g(2).

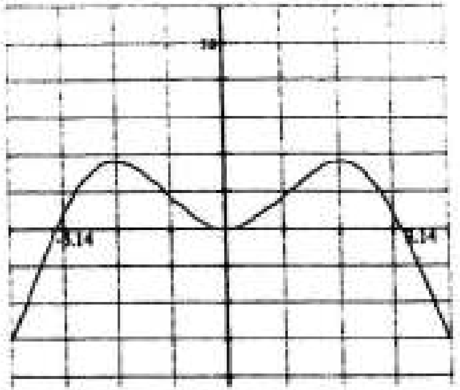
2. Find the instantaneous rate of change of g with respect to x at x=1.

3. Where is g(x) increasing? Decreasing? Justify.

4. Find the absolute minimum on [-5, 2]. Justify.

5. What are the points of inflection on the graph of g? Justify.

Given the graph of f'(x) is an even function and $\int_{-\pi}^{0} f'(x)\,dx = \pi$

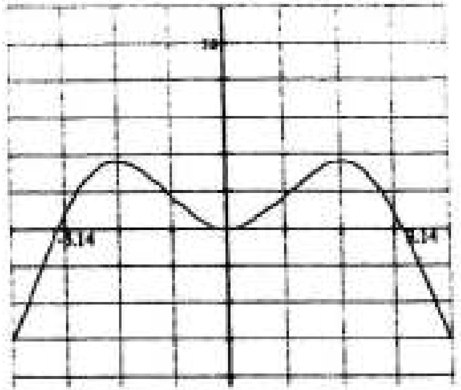


1. Approximate the following:

$$\int_{-4}^{\pi} f'(x)\,dx \qquad \int_{-4}^{0} f'(x)\,dx$$

$$\int_{-4}^{4} f'(x)\,dx \qquad \int_{-4}^{4} \left| f'(x) \right| dx \qquad \int_{-4}^{4} \left(\left| f'(x) \right| + 1 \right) dx$$

Given the graph of f'(x) is an even function and $\int_{-\pi}^{0} f'(x)\,dx = \pi$



2. Given that f(-4)=1, find:

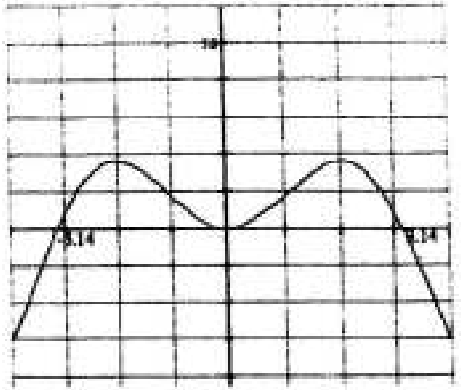
$f(-\pi)$

$f(0)$

$f(\pi)$

$f(4)$

Given the graph of f'(x) is an even function and $\int_{-\pi}^{0} f'(x)\,dx = \pi$



3. Where would f(x) have a relative maximum? Justify.

4. Give the approximate x-coordinates of the points of inflection on f(x). Justify.

5. Give the intervals where f(x) is concave up. Justify.

6. Find the absolute maximum of f(x). Justify.