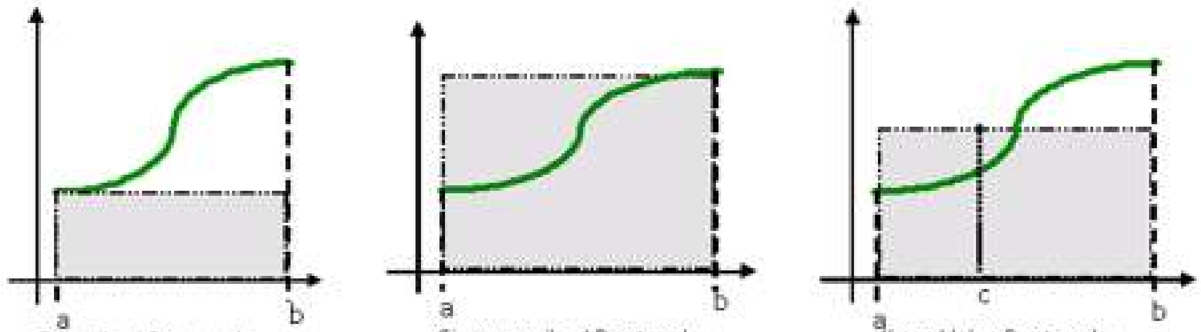
Sec 5.3:
Mean Value Theorem
&
Avg Value of a Function

We've looked at how to find area under a curve using integrals. Now we are going to look at how to find the average height of a function (or the average value of the function.)



Inscribed Rectangle:
Area is less than the actual area

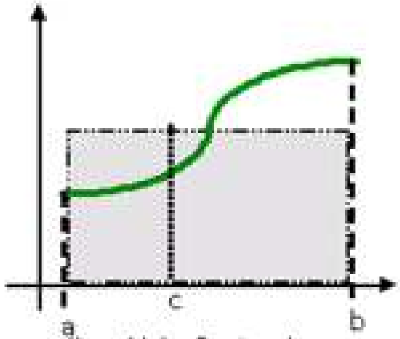
Circumscribed Rectangle:
Area is more than the actual area

Mean Value Rectangle:
Area is equal to the actual area

Let's look at this 3rd example more closely.

Area under curve using calculus = area of rectangle

$$\int_a^b f(x)dx \quad = \quad (\qquad)(\qquad)$$



Mean Value Rectangle:
Area is equal to the actual area

$$\int_a^b f(x)dx = f(c)(b-a)$$

$$\therefore f(c) =$$

This is the average value of the function!

So let's find the average value of the functions you just did to see how close your estimate was.

$q(x) = \frac{1}{2}x^2 - 2x + 3$ on $[1,4]$

$f(x) = -\cos(2x) + x + 1$ on $[0,4]$

Find the average value of the function on the given interval, then find c in the interval that gives that average value.

$$f(x) = x^2 - 2x \quad \text{on} \quad [1, 4]$$

The average value of a function is very useful in particle motion problems, but it is easy to confuse when to use it. Let's see if we can't figure it out! :)

By the FTC: $$\int_a^b v(t)dt = s(b) - s(a)$$

Now let's divide both sides by time:

What does that give us???

So there are TWO ways to find *Average Velocity*:

Given velocity

$$\frac{1}{b-a}\int_a^b v(t)dt$$

Given position

$$\frac{s(b)-s(a)}{b-a}$$

Find the average velocity over the interval [0, 3] of a particle given. . .

$v(t) = 2t$

$s(t) = t^2$

We can use the same philosophy with finding average acceleration as well.

$$\frac{1}{b-a}\int_a^b a(t)dt = \frac{v(b)-v(a)}{b-a}$$

How about this. . .

The velocity of a projectile is given by the equation below for any time [0,6] minutes.

$$v(t) = -x^2 + 3x + 4$$

What is the average accleration on [0,6]?

What is the average velocity on [0,6]?

The rate at which cars are passing at a given point on a freeway can be modeled by the equatio given below, where t is the hours after midnight, 1<t<12, and r(t) is given in a thousand cars per hour.

$$r(t) = -0.04t^3 + 0.77t^2 - 3.24t + 6.68$$

At what rate is the rate of cars increasing at 7:00 am?

What is the average rate at which the rate of cars is increasing from 4:00 am to 12 noon?

$$r(t) = -0.04t^3 + 0.77t^2 - 3.24t + 6.68$$

At what time in (4,12) is the instantaneous rate of change of the rate of cars passing a given point equal to the average rate of change of cars passing a given point?

$$r(t) = -0.04t^3 + 0.77t^2 - 3.24t + 6.68$$

What is the total number of cars that pass the given point from 4:00 am to 12 noon?

What is the average rate at which the cars pass the given point from 4:00 am to 12 noon?