Sec 6.2 & 6.3:

U-Substitution

&

Integration by Parts

So far we have looked at integrals that fit one of our derivative rules (or we've been able to manipulate the problem to make it fit.) But what if the integral is too complicated??? Glad you asked!

See if you can evaluate these integrals.

$$\int 3\left(x^2+8\right)^2(2x)\,dx \qquad \int 5\cos(5x)\,dx \qquad \int xe^x\,dx \qquad \int x\sqrt{x+1}\,dx$$

The first two integrals are chain rule problems and the last two integrals are product rule problems. So how do we "undo" these more complicated derivative rules??? Let's see.

Let's start with "undoing" a chain rule derivative. I thinks it's easiest to practice chain rule first, then undo it. Shall we begin?

$f(x) = \left(x^2 + 8\right)^2 \qquad f'(x) =$ ________________

So. . . $\int 3\left(x^2 + 8\right)^2 (2x)dx$ =

$f(x) = \sin(5x) \qquad f'(x) =$ ________________

So. . . $\int 5\cos(5x)dx$ =

It's helpful to be able to identify what parts of the integral are coming from the "outside" function and which parts are the derivatives of the "inside" functions. This will help us identify the main derivative rule being used. Try to look at each integral and see the "big picture".

$\int (\text{something})^{power}\, du \qquad \int e^{(\text{something})}\, du \qquad \int \frac{1}{(\text{something})^1}\, du$

$\int \sin(\text{something})\, du \qquad \int \cos(\text{something})\, du$

Let's try some.

$$\int 3(3x-1)^4\,dx$$

u = ______________

du = ______________

$$\int x(x^2+1)^2\,dx$$

u = ______________

du = ______________

$$\int \sqrt{2x-1}\,dx$$

u = ______________

du = ______________

Let's try some.

$$\int \cos(\pi x)\,dx$$

u = ______________

du = ______________

$$\int \cos^3 x \sin x\,dx$$

u = ______________

du = ______________

$$\int x^2 e^{x^3}\,dx$$

u = ______________

du = ______________

Let's try some more.

$$\int \frac{5}{e^{4x}}\,dx$$

u = ______________

du = ______________

$$\int \frac{1}{x+1}\,dx$$

u = ______________

du = ______________

$$\int \frac{2x+1}{x^2+x}\,dx$$

u = ______________

du = ______________

How about these.

$$\int_{0}^{\pi/7} \cot(7x)\,dx$$

u = ______________

du = ______________

$$\int_{-4}^{4} \frac{2e^{2x}}{1+e^{2x}}\,dx$$

u = ______________

du = ______________

$$\int_{0}^{\pi/2} \cos^3 x\,dx$$

u = ______________

du = ______________

Now let's look at how to undo the product rule. Here you are looking for the product of two functions, such as a polynomial and a transcendental function.

$$\int xe^x\,dx \qquad \int x^2 \sin x\,dx \qquad \int x \ln x\,dx$$

Product Rule: $\dfrac{d}{dx}[uv] =$

Integrating both sides you get: $uv =$

Rearranging: $\int u\,dv =$

An integral in the form $\int u dv$ can be rewritten as $uv - \int v du$

We hope this second form will be easier to integrate than the original problem.

Remember when choosing "u" and "dv" we will be taking the derivative of "u" and finding the antiderivative of "dv". *Choose wisely!!*

Here are two main keys to Integration by Parts:

- Recognize that no other method will work.

- *Choose u and dv correctly.* You can use LIPET to help choose u:
  - L - Log Functions
  - I - Inverse Trig Functions
  - P - Polynomial Functions
  - E - Exponential Functions
  - T - Trig Functions

Let's try some.

$$\int u\,dv = uv - \int v\,du$$

$$\int xe^{3x}dx$$

u = ________ dv = ____________

du = _______ v = ______________

$$\int x^2 \ln(x)dx$$

u = ________ dv = ____________

du = _______ v = ______________

Let's try some.

$$\int u\,dv = uv - \int v\,du$$

$$\int \ln(2x)\,dx$$

u = ________ dv = ____________

du = _______ v = ______________

$$\int \arcsin(x)\,dx$$

u = ________ dv = ____________

du = _______ v = ______________

Sometimes integration by parts needst o be done more than once. This typically happens when you choose "u" to be a polynomial whose degree is greater than one. Let's see what happens.

$$\int u\,dv = uv - \int v\,du$$

Round one: $\int x^2 e^x\,dx$

u = _______ dv = __________

du = ______ v = ___________

Round two:

u = _______ dv = __________

du = ______ v = ___________

There is a simpler way to do this! The tabular method helps keep track of "u" and all of its derivatives and "dv" and all of its antiderivatives. Let's see how it works.

$\int x^2 \sin x dx$ u = ___________ dv = ___________

Alternate Signs	"u" and its **derivatives**	"dv" and its **antiderivatives**

$\int x^2 \sin x dx$

$\int x^2 \cos(4x)\,dx$

u = ___________ dv = ___________

Alternate Signs	"u" and its **derivatives**	"dv" and its **antiderivatives**

Let's try some even tougher ones.

$$\int e^x \sin x \, dx$$

How about a definite integral?

$$\int_0^{\pi/2} x \sin x \, dx$$