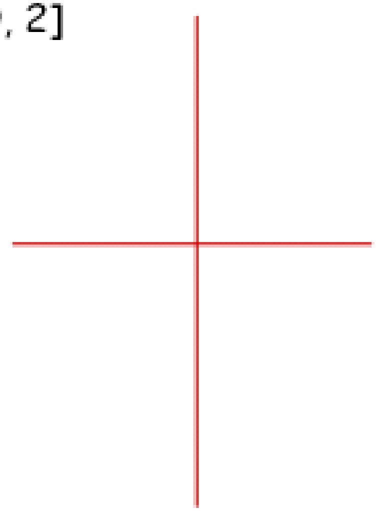
Section 4.1 & 4.2: Extreme Value Theorem and Mean Value Theorem

Does $y = x^3$ have a maximum or minimum? Can we make it have one? Let's see.

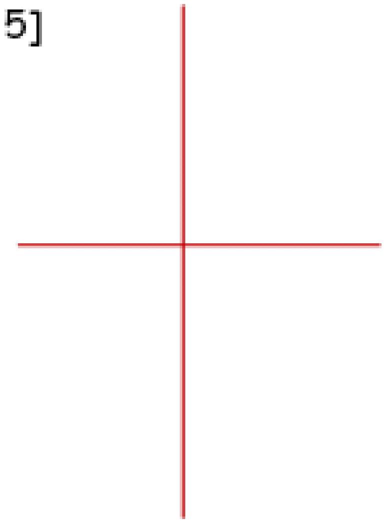
Look at the graph on the interval $(-\infty, \infty)$ Any max/mins?

How about the following intervals? Sketch a graph of the correct part of the function.

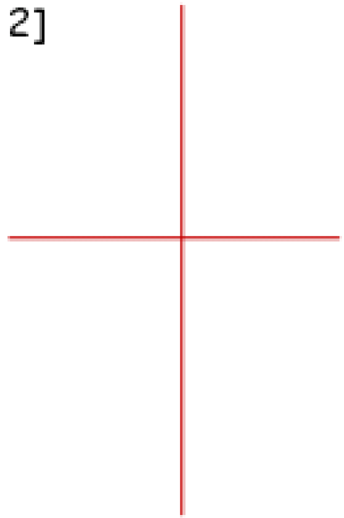
[0, 2]



[-1, 5]



[-2, 2]



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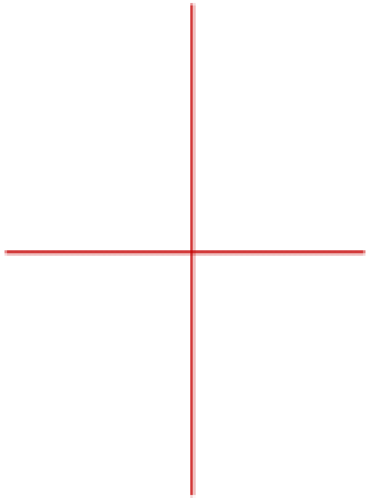
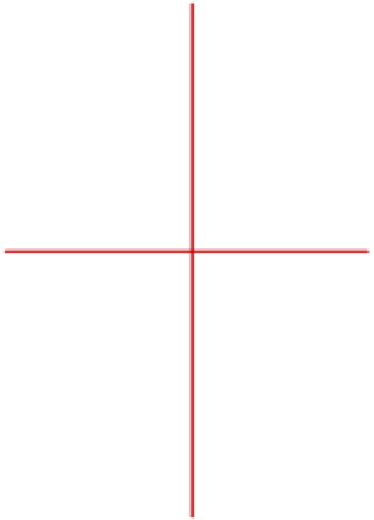
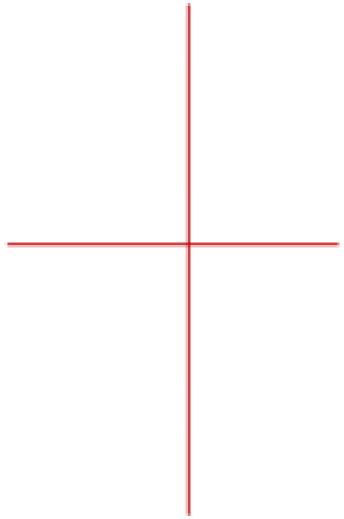


(-1, 5]



(-2, 2)



What will gaurantee that a max *and* min occur?

Extreme Value Theorem

If f is continuous on the closed interval [a,b], then f has both a minimum and a maximum on the closed interval [a,b].

Critical number - where $f'(c) = 0$ or $f'(c)$ is undefined (indicate locations of *possible* extrema)

To Find Extrema on a Closed Interval

1. Find the critical numbers of f in (a,b).
2. Evaluate f at each critical number in (a,b).
3. Evaluate f at each endpoint of $[a,b]$.
4. The least of these values is the minimum.
5. The greatest is the maximum.

Find the critical numbers and identify the absolute extrema on the given closed interval.

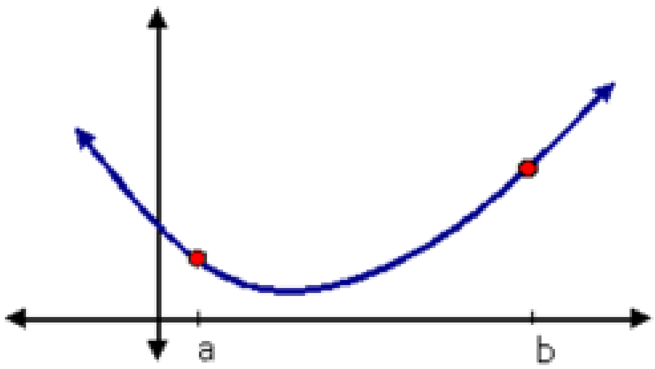
$f(x) = x^3 - 12x$ on [0, 5]

$f(x) = \ln(x+1)$ on [2, 3]

Find the critical numbers and identify the absolute extrema on the given closed interval.

$f(x) = e^{-x^2}$ on [-2, 2]

$f(x) = 2\cos x - x$ on $\left[0, \frac{3\pi}{2}\right]$



Draw a secant line through f(a) and f(b) . Can you draw a tangent line that is parallel to the secant line?

The Mean Value Theorem

If f is continuous and differentiable on an interval then there must be a place where

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

(Calculus slope = Algebra slope)

Apply the Mean Value Theorem to the function on the indicated interval.

$f(x) = x^2 - 2x \qquad [0, 3]$

$f(x) = 2\cos x \qquad \left[0, \frac{\pi}{2}\right]$

Rolle's Theorem

If f is continuous and differentiable on an interval and $f(a) = f(b)$
then there must be a place where $f'(c) = 0$

(If the height of the function at a equals the height of the function at b, then there is an extrema somewhere between a and b.)

Determine whether Rolle's Theorem applies over the interval, and, if so, find all c's so that f'(c)=0.

$f(x) = x - \sqrt{x}$ $\quad [0,1]$

$f(x) = |x| - 2$ $\quad [0,4]$

Given $f(x)$ is differentiable on [-1,3] and $f(-1)=-4$ and $f(3)=12$, which of the following (if any) must be true?

I. $f(x)$ has a zero in [-1,3].

II. $f(x)$ has a critical number in [-1,3].

III. $f'(x)=4$ in [-1,3].