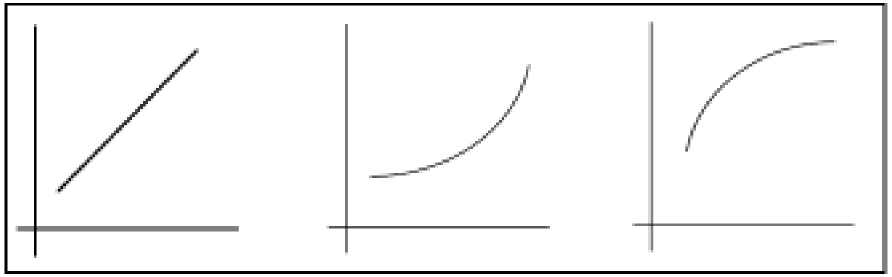
# Section 4.3: Curve Sketching

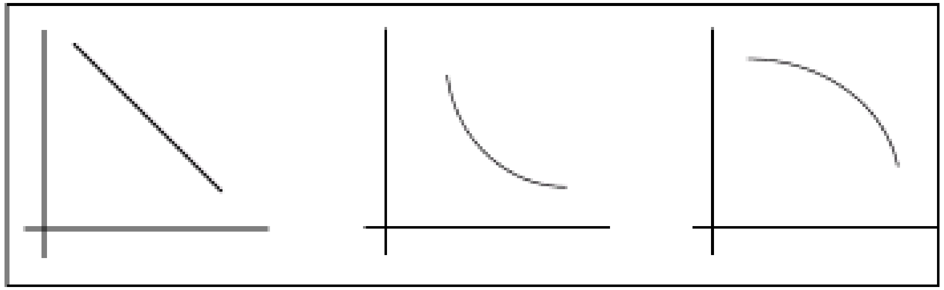
Increasing curves come in 3 varieties as shown. In each case, draw tangent lines at several points along the curves.



What is true about the slope of each tangent line above? ____________________

So. . . if f'(x) ______ 0, then we know that f(x) is ____________________.

Decreasing curves come in 3 varieties as shown. In each case, draw tangent lines at several points along the curves.



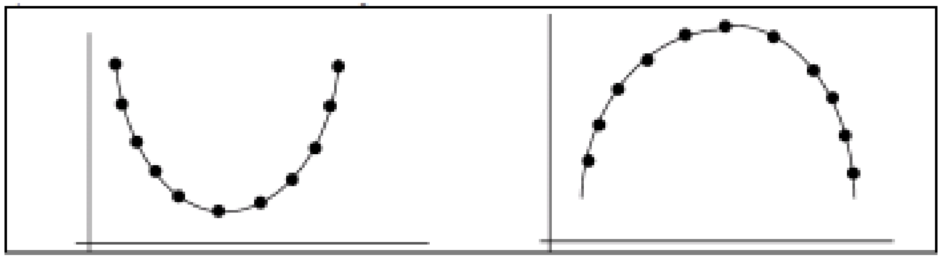
What is true about the slope of each tangent line above? ________________________

So. . . if $f'(x)$ ______ 0, then we know that $f(x)$ is _________________________.

When we examine the functions that have curves to them, we will define their curvature in terms of concavity. Concavity comes in two flavors - concave up (like a cup) and concave down (like a frown).

Draw tangent lines at the points shown. What is true about the slopes of the lines as you go left to right?

The slopes are ______________



The slopes are ______________

Now we are talking about the *rate of change* of the *slope of the tangent line*.

Whew! What derivative is that?????

So let's see. . . if the slopes are getting bigger, then f'(x) is ______________, which means that the slopes of f'(x) will be ____________, which means that f''(x) _____ 0.

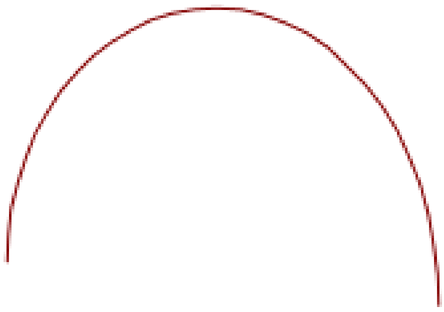
How about this. . . if the slopes are getting smaller, then f'(x) is ______________, which means that the slopes of f'(x) will be _______________, which means that f''(x) _____ 0.

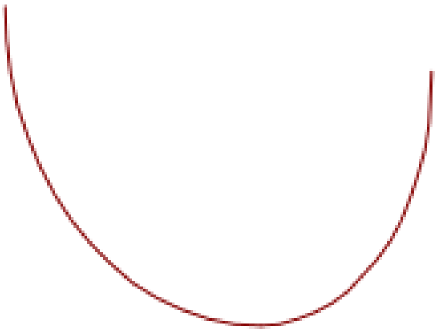
Let's see if we can put it all together before I lose you!

Fill in the chart describing the attributes of f(x) based on f'(x) and f''(x).

| | $f'(x) > 0$ | $f'(x) < 0$ | $f'(x) = 0$ |
|---|---|---|---|
| $f''(x) > 0$ | | | |
| $f''(x) < 0$ | | | |
| $f''(x) = 0$ | | | |

Let's look at max and mins quickly.  Draw in tangent lines on each side of the extrema and then complete the statement.





The curve switches from

________ to _________ so

f'(x) must change from _____ to ______.

The curve switches from

________ to _________ so

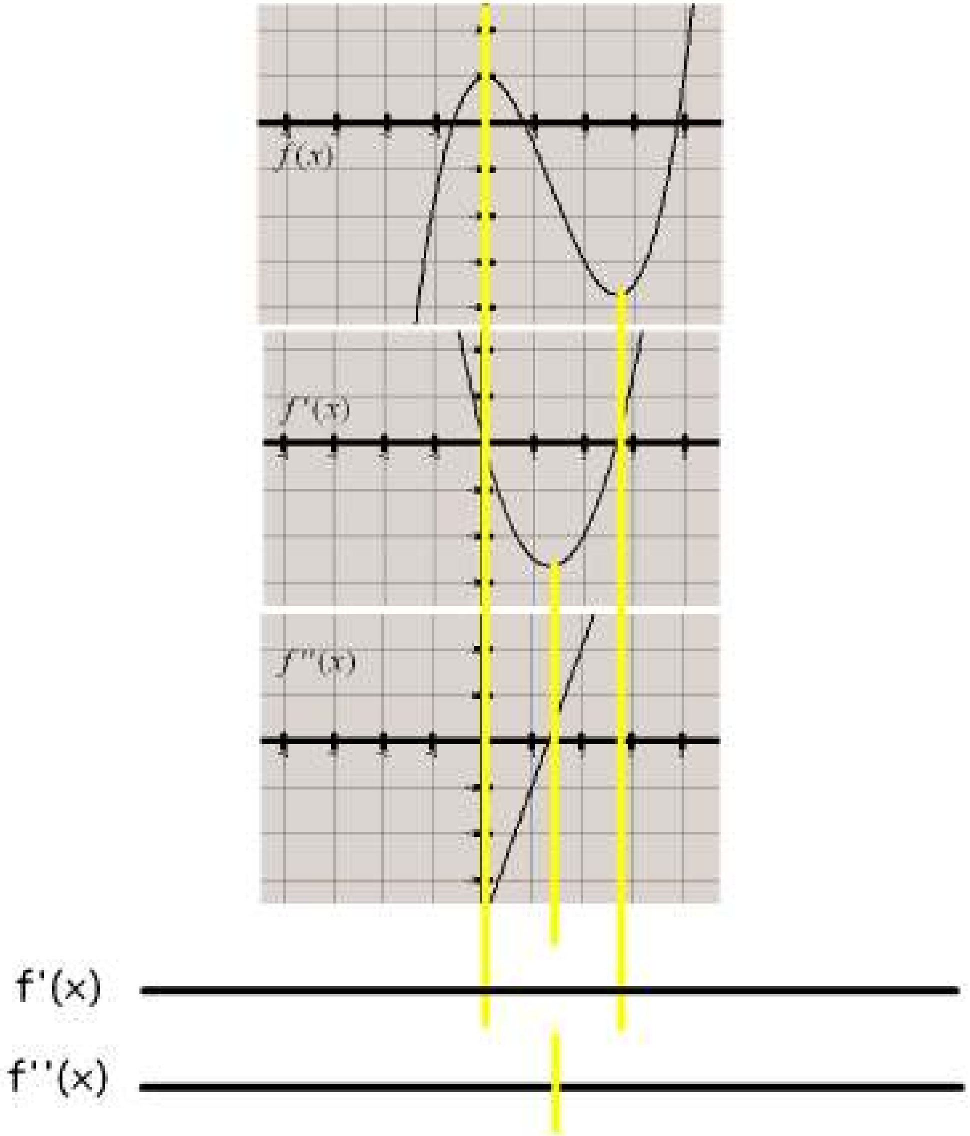
f'(x) must change from _____ to ______.

Here is what you must memorize!!! (This would be a good quiz for next time :)

**Graphical Analysis: Increasing/Decreasing Intervals and Concavity**

- Critical number occur when $f'(x) = 0 / und$
- $f(x)$ is increasing when $f'(x)$ is $+$
- $f(x)$ is decreasing when $f'(x)$ is $-$
- Relative Max occurs when $f'(x)$ changes from $+/-$
- Relative Min occurs when $f'(x)$ changes from $-/+$
- Possible points of inflection occur when $f''(x) = 0 / und$
- $f(x)$ is concave up when $f''(x)$ is $+$
- $f(x)$ is concave down when $f''(x)$ is $-$
- Points of inflection only occur when $f(x)$ changes concavity

Here's another way to look at it.



Analytcally determine intervals for increasing/decreasing, concave up/concave down and all extrema.

$$f(x) = x^3 - 3x^2 + 2$$

Critical #s, inc/dec, extrema

Points of inflection, concavity

Analytcally determine intervals for increasing/decreasing, concave up/concave down and all extrema.

$$f(x) = x^4 - 2x^2$$

Critical #s, inc/dec, extrema

Points of inflection, concavity

Analytcally determine intervals for increasing/decreasing and all extrema (do not do concavity).

$$f(x) = \frac{x^2 + 1}{x^2 - 9}$$

# Section 4.3
# Day 2
# More Curve Skethcing

Analytcally determine intervals for increasing/decreasing, concave up/concave down and all extrema.

$$f(x) = -\cos x - \frac{1}{2}x \qquad [0, 2\pi]$$

Critical #s, inc/dec, extrema

Points of inflection, concavity

Analytcally determine intervals for increasing/decreasing, concave up/concave down and all extrema.

$$f(x) = e^{-x^2}$$

Critical #s, inc/dec, extrema

Points of inflection, concavity